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The bfd-norm on spaces of analytic functions and the numerical range

Abstract. Let E be the open region in the complex plane bounded by an ellipse. The B. and F. Delyon norm $\|\cdot\|_{\mathrm{bfd}}$ on the space $\mathrm{Hol}(E)$ of holomorphic functions on E is defined by

$$\|f\|_{\mathrm{bfd}} \stackrel{\mathrm{def}}{=} \sup_{T \in \mathcal{F}_{\mathrm{bfd}}(E)} \|f(T)\|,$$

where $\mathcal{F}_{\mathrm{bfd}}(E)$ is the class of operators T such that the closure of the numerical range $W(T)$ of T is contained in E. The name of the norm recognizes a celebrated theorem of the brothers Delyon, which implies that $\|\cdot\|_{\mathrm{bfd}}$ is equivalent to the supremum norm $\|\cdot\|_\infty$ on $\mathrm{Hol}(E)$.

We show that there exists an interesting connection between the bfd norm on $\mathrm{Hol}(E)$ and the supremum norm $\|\cdot\|_\infty$ on the space $\mathrm{H}^\infty(G)$ of bounded holomorphic functions on the symmetrized bidisc, the domain G in $\mathbb{C}^2$ defined by

$$G \stackrel{\mathrm{def}}{=} \{(z+w, zw) : |z| < 1, |w| < 1\}.$$

It transpires that there exists a holomorphic embedding $\tau : E \to G$ having the property that, for any bounded holomorphic function f on E,

$$\|f\|_{\mathrm{bfd}} = \inf\{\|F\|_\infty : F \in \mathrm{H}^\infty(G), F \circ \tau = f\},$$

and moreover, the infimum is attained at some $F \in \mathrm{H}^\infty(G)$.

We also consider connections between operators T with the closure of $W(T)$ contained in E and Douglas-Paulsen operators.

The talk is based on joint work with Jim Agler and Nicholas Young [1].

[1] J. Agler, Z. A. Lykova and N. J. Young, On the operators with numerical range in an ellipse, *J. Funct. Anal.*, **287**(8) Article 110556 (2024) https://doi.org/10.1016/j.jfa.2024.110556.