Desingularized models and directional derivatives for Schur-Agler class functions

Connor Evans, Newcastle University Functional Analysis Seminar 2025

We describe a generalization of the notion of a Hilbert space model of a function φ in the Schur-Agler class of the polydisc $\mathbb{D}^d$. This generalization is well adapted to the investigation of boundary behaviour of φ at a mild singularity τ on the d-torus $\mathbb{T}^d$. We prove the existence of a generalized model with an enhanced continuity property at such a singularity τ. We use this result to prove the directional differentiability of a function φ in the Schur-Agler class at a singular point on the d-torus for which the Carathéodory condition holds and to calculate the corresponding directional derivative. The results of this talk extend to the polydisc results of Agler, McCarthy, Tully-Doyle and Young which generalized to the bidisc the classical Julia-Wolff-Carathéodory theorem about analytic self-maps of the unit disc $\mathbb{D}$.

This talk is based on joint work with Jim Agler, Zinaida Lykova and Nicholas Young [1].

References

[1] J. Agler, C. Evans, Z. A. Lykova and N. J. Young, Boundary behavior of functions in the Schur-Agler class of the polydisc, arxiv.org/abs/2508.13742, 2025.