# Localized States in Rotating Convection at Low Prandtl Number

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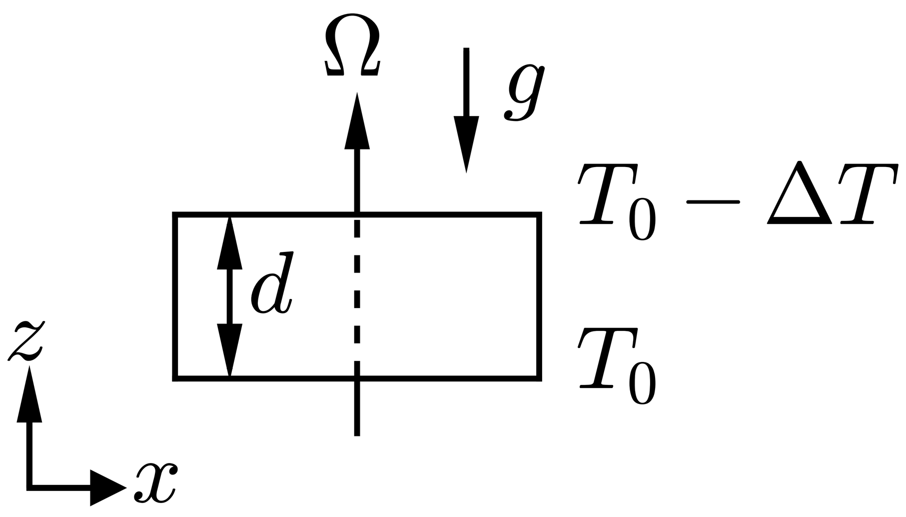
## Introduction

We are interested in the formation of large-scale patterns in rapidly-rotating convection, such as the convective nests observed in 3D spherical convection (Grote & Busse, 2001; Brown et al., 2008), which might explain why some planets such as Mercury generate weak dipolar fields (Heimpel et al., 2005). We consider the most basic setup where localized convective structures are observed (Beaume et al., 2013) and study the motion of a layer of fluid in a rotating box, uniformly heated from below. We also consider the system at low Prandtl numbers ($\mathrm{Pr} = 0.025$), relevant to planetary cores.

## Governing Equations

We assume that all variations in the y-direction vanish and so the governing equations are the two-dimensional Boussinesq equations. The geometry of the system can be seen below,




with boundary conditions, $\psi = 0$, $\partial^2\psi/\partial z^2 = 0$, $T = 0$ and $\partial v/\partial z = 0$ at $z = 0, d$.

The two dimensional Boussinesq equations in dimensionless form are then,

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial \eta}{\partial t} - \mathcal{J}(\psi, \eta)\right) = \sqrt{\mathrm{Ta}}\frac{\partial v}{\partial \mathrm{z}} - \mathrm{Ra}\frac{\partial T}{\partial \mathrm{x}} + \nabla^2\eta$$

$$\frac{\partial T}{\partial t} - \mathcal{J}(\psi, T) = \nabla^2 T - \frac{\partial \psi}{\partial \mathrm{x}}$$

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial v}{\partial t} - \mathcal{J}(\psi, v)\right) = \nabla^2 v - \sqrt{\mathrm{Ta}}\frac{\partial \psi}{\partial \mathrm{z}}$$

with non-dimensional parameters $\mathrm{Pr} = \nu/\kappa$ (Prandtl number), $\mathrm{Ta} = 4\Omega^2 d^4/\nu^2$ (Taylor number) and $\mathrm{Ra} = g\alpha\Delta T d^3/\kappa\nu$ (Rayleigh number) where $\psi$ is the streamfunction, $\eta = \nabla^2\psi$, $v$ is the zonal velocity and $T$ is the temperature deviation from the linear profile.

The vertical boundaries are flat, stress-free and perfectly conducting and horizontally the boundaries are periodic. We use a simplified model where a minimal number of Fourier modes are retained in the vertical direction whilst keeping the full horizontal structure (Blanchflower, 1999). This is motivated by the simple vertical structure of the convective flow at the linear onset of convection (Chandrasekhar, 1953).

$$\psi = A(t,x)\sin(\pi z) + B(t,x)\sin(2\pi z)$$

$$T = C(t,x)\sin(\pi z) + D(t,x)\sin(2\pi z)$$

$$v = E(t,x) + F(t,x)\cos(\pi z) + G(t,x)\cos(2\pi z)$$

leaving us with a system of partial differential equations for the coefficients, $A(t,x), \ldots, G(t,x)$ which we solve pseudo-spectrally.

## References


Grote & Busse (2001) *FDR* **28** 349-368

Brown et al. (2008) *APJ* **689**(2) 1354

Heimpel et al. (2005) *EPSL* **236** 542-557

Beaume et al. (2013) *JFM*, **717** 417-448

Blanchflower (1999) *PLA*, **261**(1) 74-81

Chandrasekhar (1953) *PRSLA*, **217**(1130) 306-327

Cox & Matthews (2001) *Physica D*, **149** 210-229


## Numerical Results

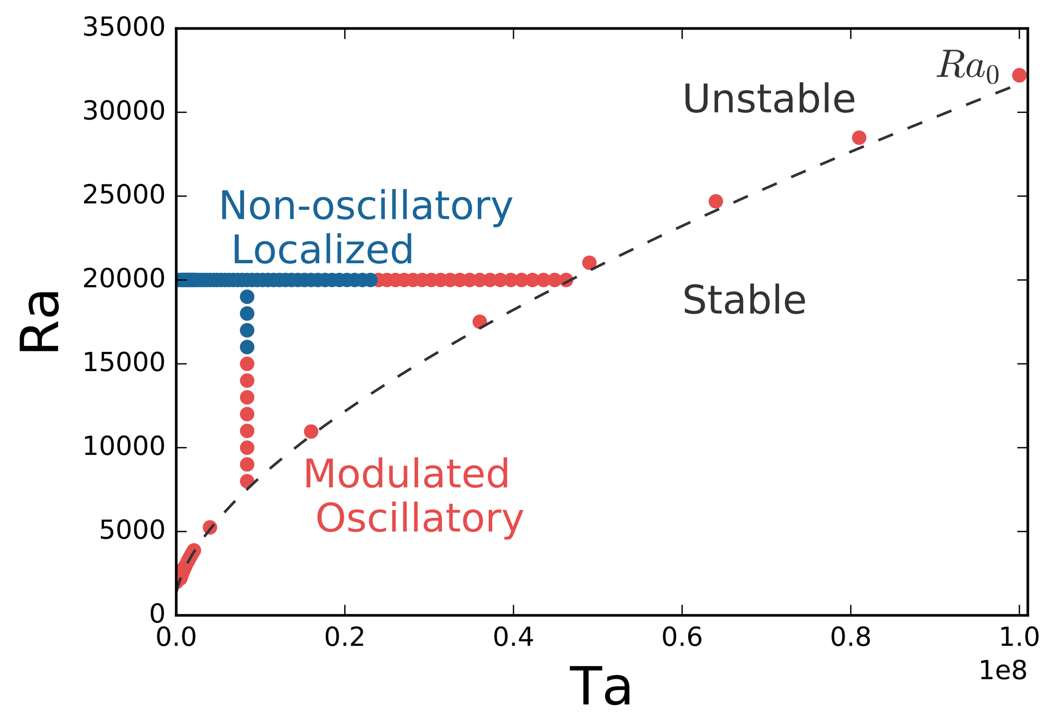



**Figure 1:** Plot of the full parameter space explored at fixed Prandtl number, $\mathrm{Pr} = 0.025$. $\mathrm{Ra}_0 = 2(\mathrm{Pr}+1)\alpha^{-2}[(\alpha^2+1)^3\pi^4 + \mathrm{Pr}^2\mathrm{Ta}/(\mathrm{Pr}+1)^2]$ is the critical Rayleigh number for the onset of overstable motions where $\alpha$ is the critical horizontal wavenumber (Chandrasekhar, 1953).

### 1 Modulated Oscillatory States

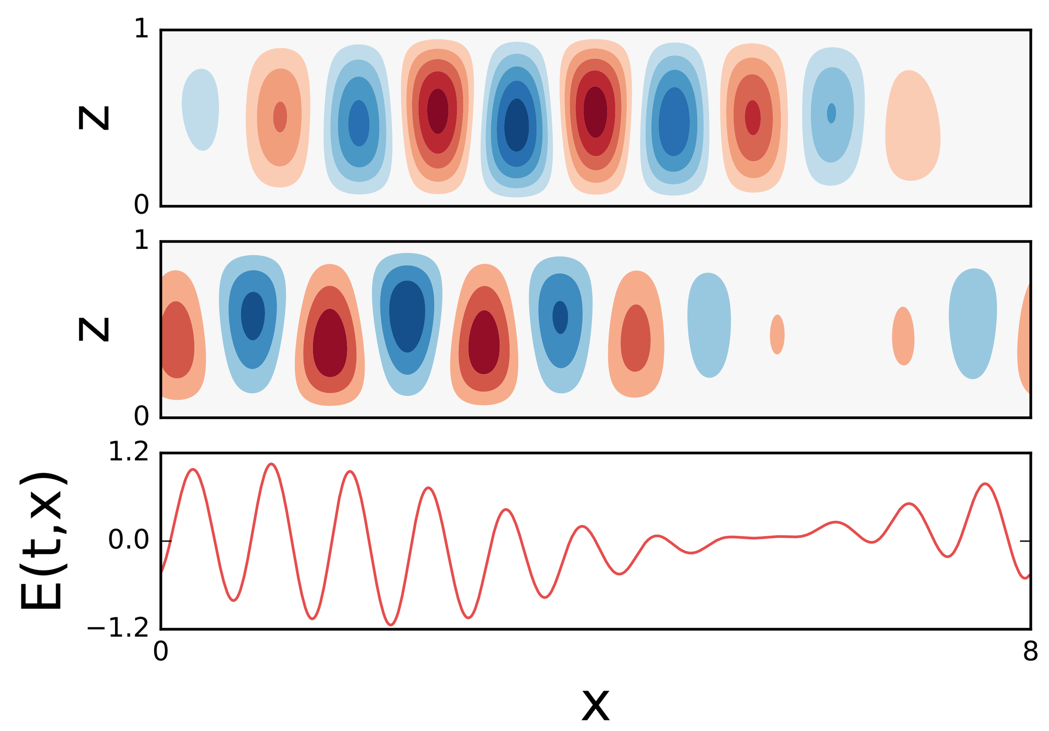



**Figure 2:** Contour plots of the streamfunction, $\psi$ *(top)*, temperature, $T$ *(middle)* and plot of the vertically averaged zonal velocity, $E(t,x)$ *(bottom)* for $\mathrm{Pr} = 0.025$, $\mathrm{Ta} = 8.4 \times 10^6$, $\mathrm{Ra} = 8 \times 10^3 \approx 1.06\mathrm{Ra}_0$.

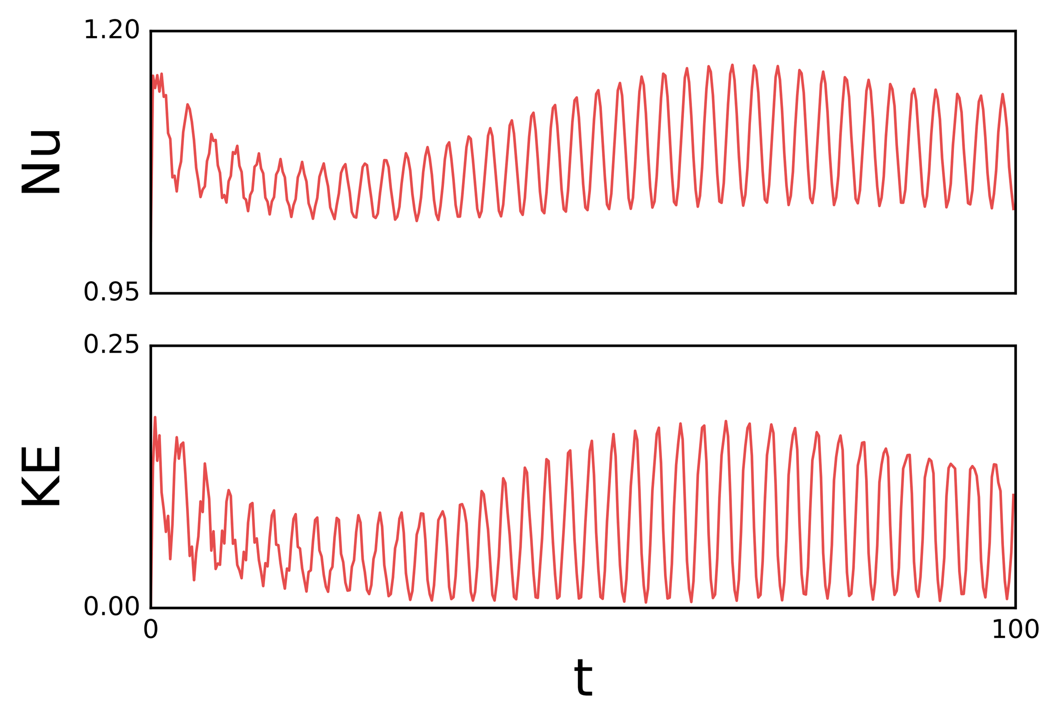



**Figure 3:** Time series of the Nusselt number, Nu and kinetic energy, KE on the thermal diffusion timescale corresponding to Figure 2.

### 2 Localized Non-oscillatory States

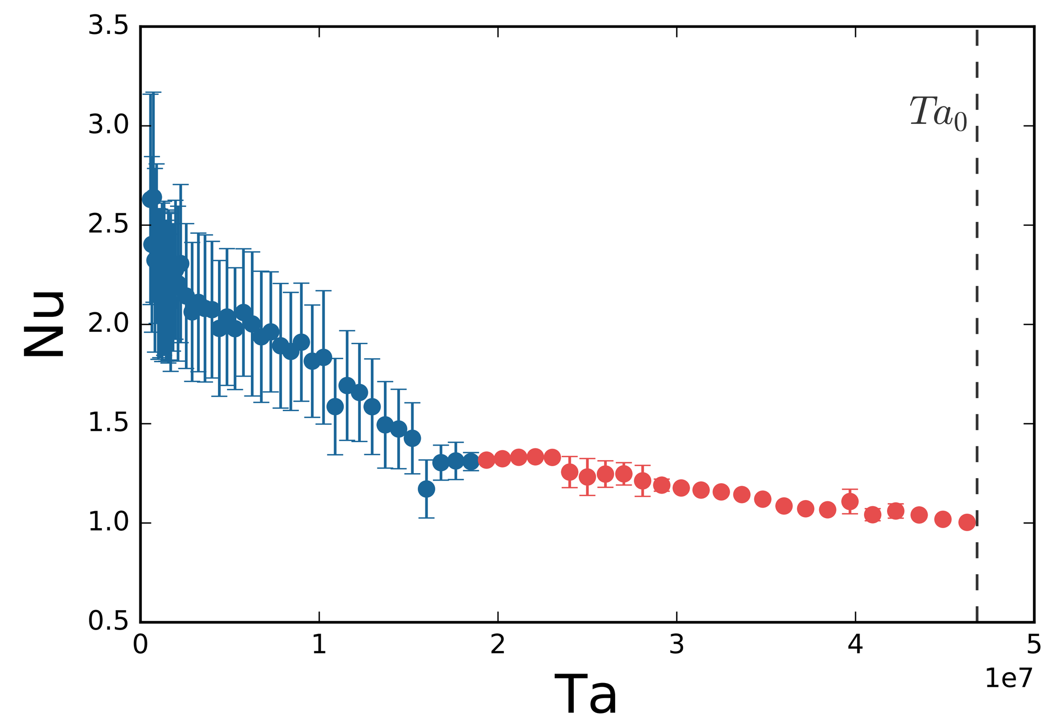



**Figure 4:** Nusselt number, Nu as a function of Ta for constant $\mathrm{Ra} = 2 \times 10^4$. The critical Taylor number such that $\mathrm{Ra}_0 = 2 \times 10^4$ is $\mathrm{Ta}_0 \approx 4.68 \times 10^7$. The solution eventually drops onto the modulated oscillatory branch (red).

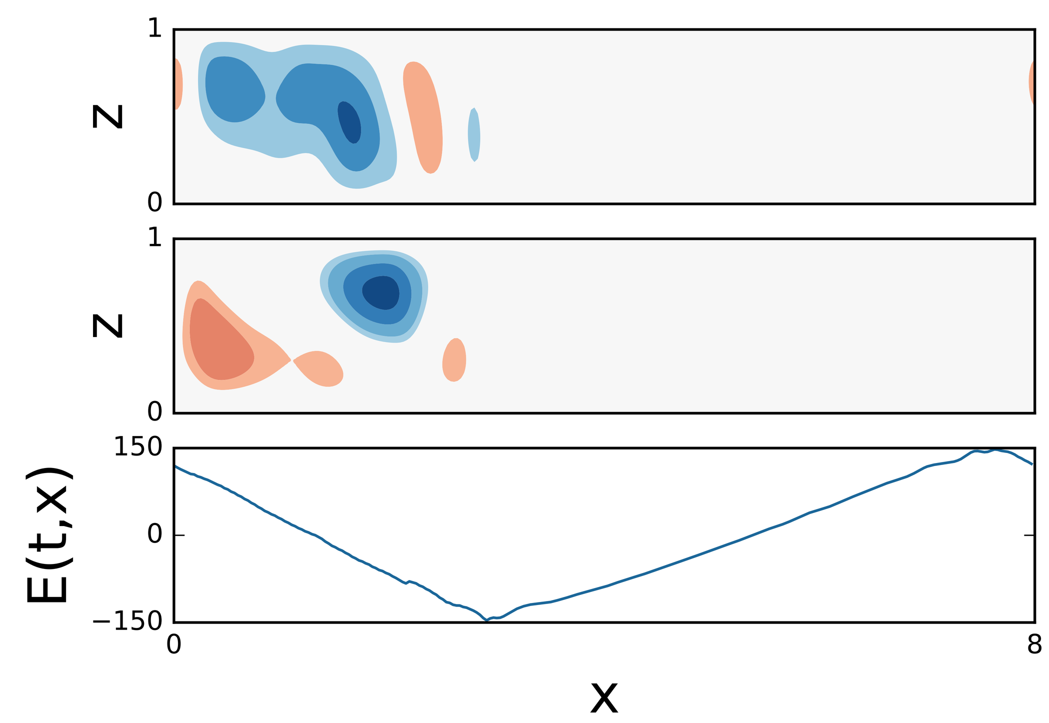



**Figure 5:** Contour plots of the streamfunction, $\psi$ *(top)*, temperature, $T$ *(middle)* and plot of the vertically averaged zonal velocity, $E(t,x)$ *(bottom)* for $\mathrm{Pr} = 0.025$, $\mathrm{Ta} = 1.5 \times 10^7$, $\mathrm{Ra} = 2 \times 10^4 \approx 1.9\mathrm{Ra}_0$. If we calculate the effective Taylor number in the cyclonic region we find that where the convection is suppressed, $\mathrm{Ta}_{\mathrm{eff}} \approx 5.09 \times 10^7 > \mathrm{Ta}_0$ and so the effective Taylor number is subcritical.

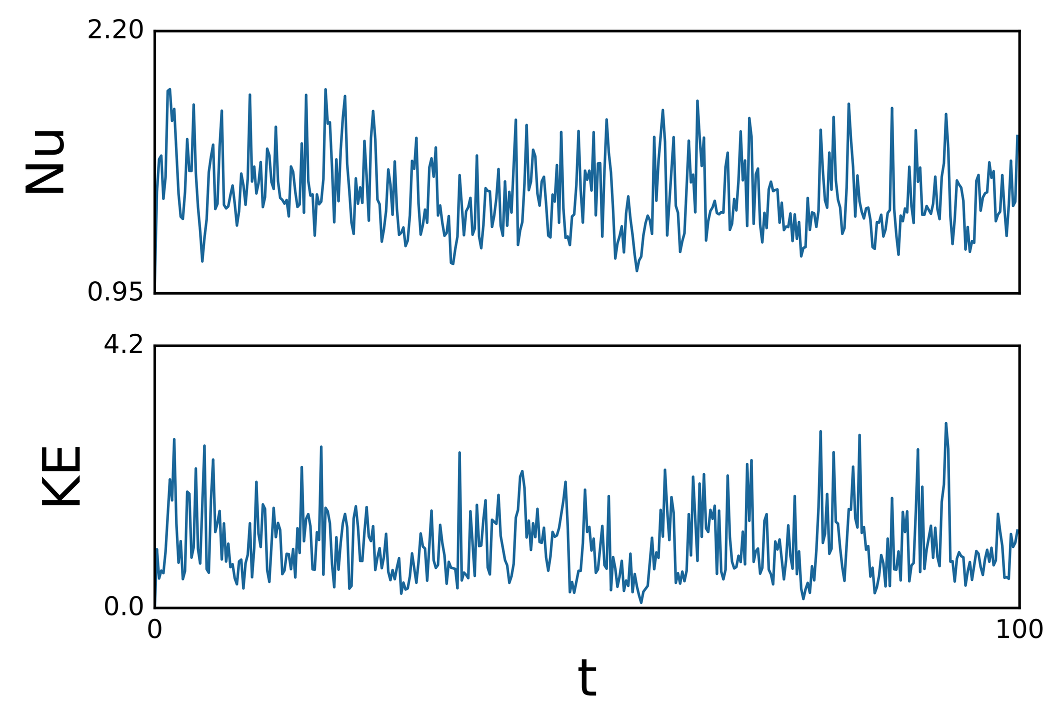



**Figure 6:** Time series of the Nusselt number, Nu and kinetic energy, KE on the thermal diffusion timescale corresponding to Figure 5.

## Conclusions

- Localized and modulated convective states can be found at low Prandtl numbers.
- Just above the oscillatory onset for convection, modulated states are observed which may be due to large-scale wave modes relating to an underlying conservation law (Cox & Matthews, 2001).
- In the highly supercritical regime, localized convective states are observed. In this regime the zonal flow is strong and inhibits convection in the region with cyclonic vorticity.