Localized states in rotating convection at low Prandtl number

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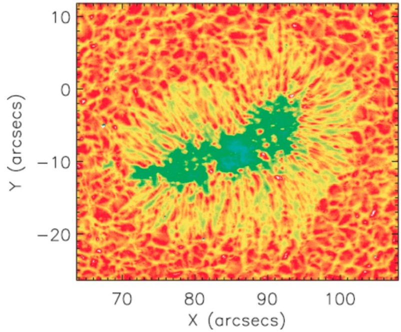
Motivation

• Strong vertical magnetic fields cause darkened regions inside sunspots called umbrae.

• Bright localized features known as umbral dots are observed.

• Locally enhanced convection in rapidly rotating stars (E.g. Brown et al, 2008).



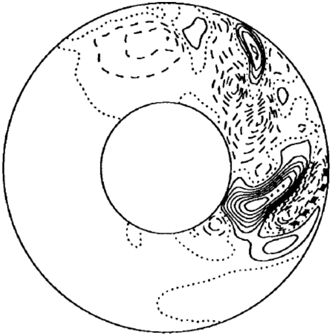


Figure: Left: Kitai et al. (2007). Right: Grote & Busse (2001)

• Steady localized states were found in 2D magnetoconvection in Blanchflower (1999).

• Convective eddies expel magnetic flux (Weiss, 1966).

• To conserve magnetic flux, this must build up elsewhere, suppressing convection away from localized regions.

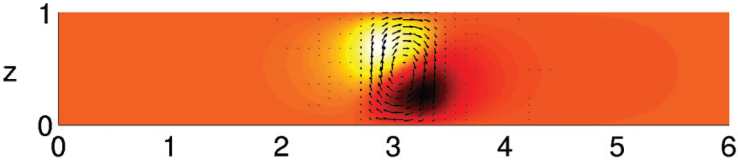
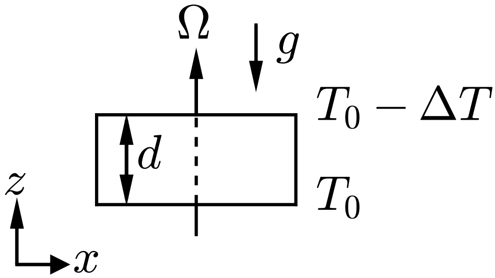


Figure: Buckley & Bushby (2013)

Geometry

- Stress-free, fixed-temperature upper/lower boundaries.
- Periodic horizontal boundaries.
- Invariance in the y-direction.
- Constant rotation, $\boldsymbol{\Omega}$, aligned with gravity.



Dimensionless parameters:

$$\mathrm{Pr} = \frac{\nu}{\kappa}, \quad \mathrm{Ta} = \frac{4\Omega^2 d^4}{\nu^2}, \quad \mathrm{Ra} = \frac{g\alpha\Delta T d^3}{\kappa\nu}. \tag{1}$$

2D dimensionless Boussinesq equations

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial\eta}{\partial t}-\mathcal{J}(\psi,\eta)\right)=\sqrt{\mathrm{Ta}}\frac{\partial v}{\partial z}-\mathrm{Ra}\frac{\partial T}{\partial x}+\nabla^2\eta, \tag{2}$$

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial v}{\partial t}-\mathcal{J}(\psi,v)\right)=-\sqrt{\mathrm{Ta}}\frac{\partial\psi}{\partial z}+\nabla^2 v, \tag{3}$$

$$\frac{\partial T}{\partial t}-\mathcal{J}(\psi,T)=-\frac{\partial\psi}{\partial x}+\nabla^2 T, \tag{4}$$

where $\eta=\nabla^2\psi$ is the y-component of vorticity.
Critical Taylor number, Ta_0. $\mathrm{Ta}>\mathrm{Ta}_0$: convection is suppressed, $\mathrm{Ta}<\mathrm{Ta}_0$: convection onsets as oscillatory motions.

Vertically truncated model

Vertical structure of the convective flow is **simple** at the linear onset of convection (Chandrasekhar, 1953).
Use a **minimal number of Fourier modes in the vertical direction**, allowing the system to be nonlinear, keeping the full horizontal structure.

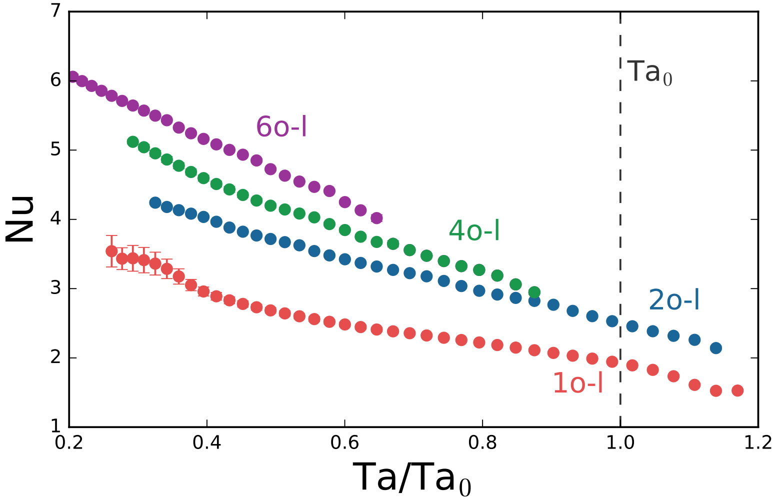
$$\psi = A(t,x)\sin(\pi z), \tag{5}$$

$$T = B(t,x)\sin(\pi z) + C(t,x)\sin(2\pi z), \tag{6}$$

$$v = D(t,x) + E(t,x)\cos(\pi z). \tag{7}$$

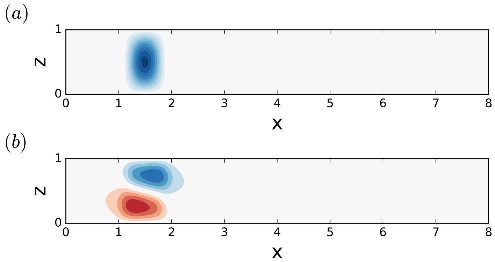
Solve the resulting PDEs pseudospectrally with an AB2 time-stepping method.
Simulations at **low Prandtl** number and **fixed Rayleigh** number, **varying Taylor** number.

$Pr = 0.025$, $Ra = 2 \times 10^4$ Results

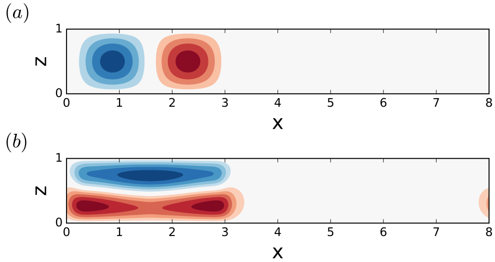


Oscillatory localized states

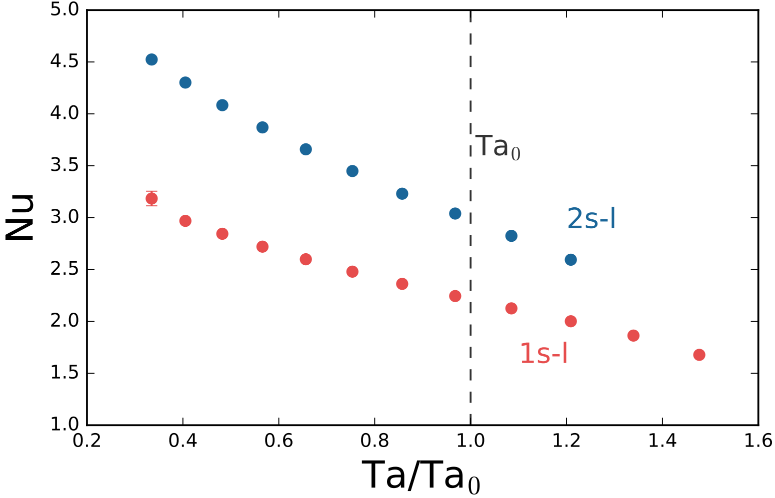
Ta $\approx 1.11\text{Ta}_0$:



Ta $\approx 0.6\text{Ta}_0$:

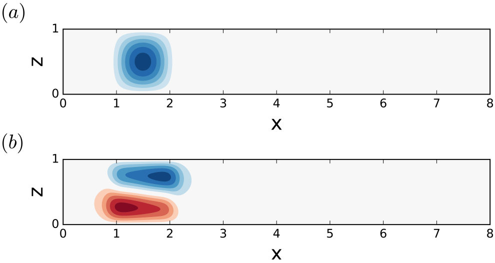


$Pr = 0.1$, $Ra = 2 \times 10^4$ Results

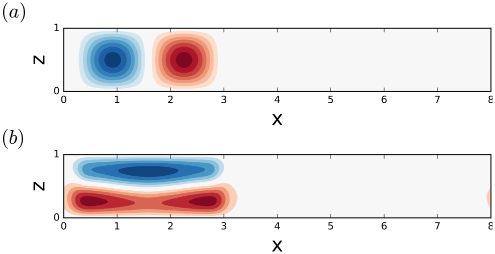


Steady localized states

Ta ≈ 1.21Ta$_0$:

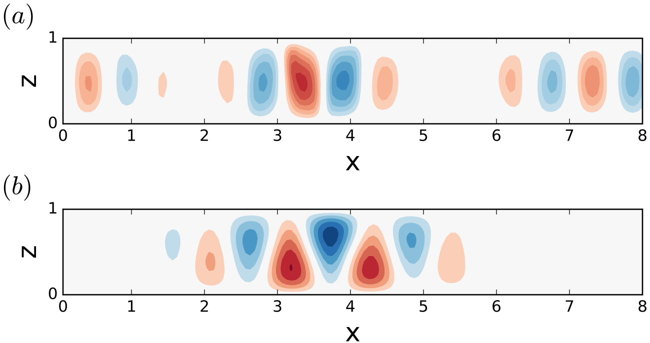


Ta ≈ 0.6Ta$_0$:



Fully resolved model

We modify the code in Cattaneo et al. (2003), which solves the 3D MHD equations in the Rayleigh-Bénard convection setup for Boussinesq fluids in a local, Cartesian geometry. We solve the system in 2D by setting all derivatives in the y-direction to zero.



$\mathrm{Pr} = 0.025, \ \mathrm{Ra} = 2 \times 10^4, \ \mathrm{Ta} \approx 0.6\mathrm{Ta}_0$

Conclusions & Further Work

Conclusions:

- In a vertically truncated model we observe both oscillatory and steady localized states.
- Convection is suppressed in regions of cyclonic vorticity where the effective Taylor number is subcritical.
- In the vertically truncated model, subcritical oscillatory and steady states exist.

Further Work:

- Continue to explore the regimes with the fully resolved 2D code.
- Explore subcritical dynamos.