Cartesian convection driven dynamos

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- Many planets have magnetic fields which are generated by a dynamo, such as in the Earth.
- In magnetohydrodynamics (MHD), convection in a rotating, electrically conducting fluid acts to maintain a magnetic field.
- This fluid can be driven by gradual cooling in the interior of the planet.
- This can happen if the convection is able to produce a magnetic field strong enough to alter the structure of the convective flows.

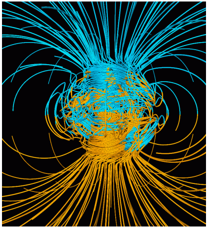


Figure: Glatzmaier & Roberts (1995)

- Some planets such as Mars do not presently have a magnetic field, but show evidence of having one in the past.
- Rocks on the surface show strong remnant magnetisation (Acuña et al., 1999).
- Studies observe that the cessation of the Martian dynamo occured rapidly (Lillis et al., 2008).
- One possible cause of this sudden termination is subcritical dynamo action.

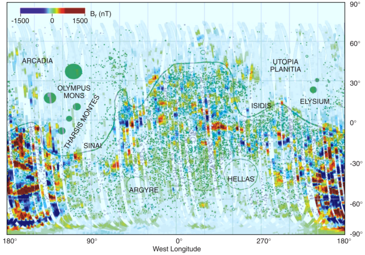
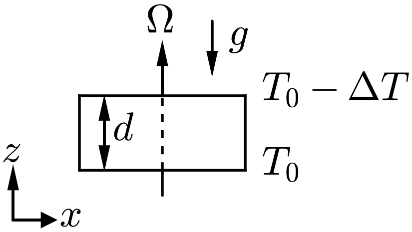


Figure: Acuña et al. (1999)

Model

- Aim to reproduce and extend the results of Stellmach & Hansen (2004).
- Convection-driven dynamo simulations in a rotating plane layer.
- Electrically conducting Boussinesq fluid.
- Constant rotation, $\mathbf{\Omega}$, aligned with gravity.
- Periodic boundaries in (x, y), stress-free, impermeable boundaries in z.
- Magnetic boundary conditions are electrically insulating.



Governing Equations

$$\frac{\mathrm{Ek}}{q\mathrm{Pr}}(\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}) - \boldsymbol{B}\cdot\nabla\boldsymbol{B} = \mathrm{Ek}\nabla^2\boldsymbol{u} - \nabla\Pi - \hat{\boldsymbol{z}}\times\boldsymbol{u} + q\mathrm{Ra}\,T\hat{\boldsymbol{z}}, \quad (1)$$

$$\partial_t \boldsymbol{B} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} - \boldsymbol{B}\cdot\nabla\boldsymbol{u} = \nabla^2\boldsymbol{B}, \quad (2)$$

$$\partial_t T + \boldsymbol{u}\cdot\nabla T = q\nabla^2 T, \quad (3)$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad (4)$$

$$\nabla\cdot\boldsymbol{B} = 0. \quad (5)$$

Dimensionless parameters:

$$\mathrm{Pr} = \frac{\nu}{\kappa}, \quad q = \frac{\kappa}{\eta}, \quad \mathrm{Ra} = \frac{g\alpha\Delta T d^3}{\kappa\nu}, \quad \mathrm{Ek} = \frac{\nu}{2\Omega d^2}. \quad (6)$$

Subcritical Dynamos

• Subcritical dynamo action is dynamo action for convective forcing below the threshold necessary for convective motions to occur in the absence of magnetic fields.

• The energy required to sustain the dynamo is far less than required to initiate the dynamo in the absence of the strong magnetic field.

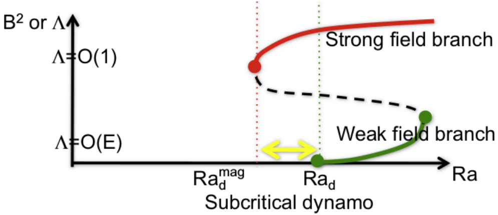


Figure: Hori & Wicht (2013)

Moderately Supercritical Dynamos

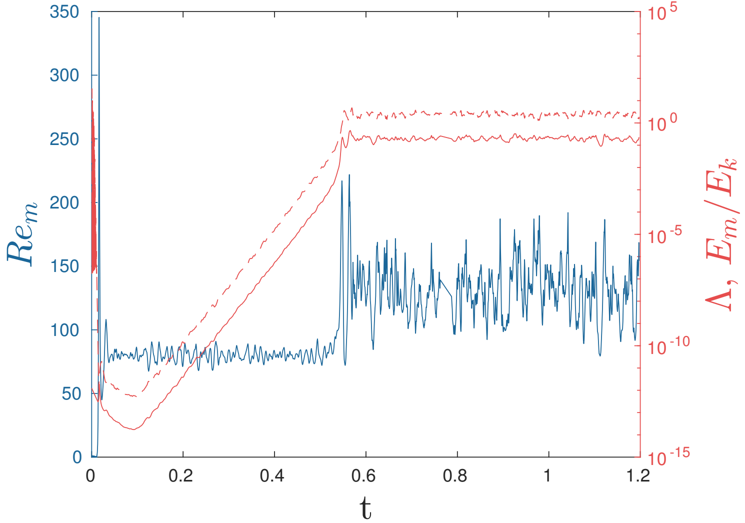


Figure: Pr $= 1$, $q = 1$, Ek $= 5 \times 10^{-6}$, Ra/Ra$_c = 1.18$.

Flow Structures

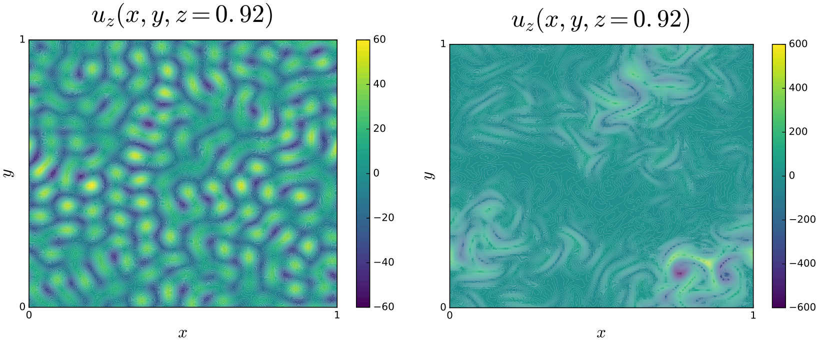
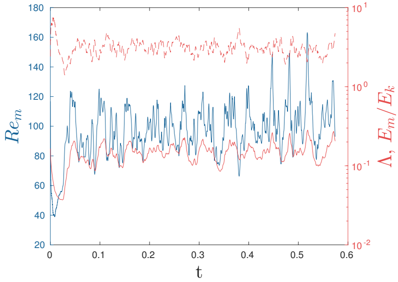


Figure: (Left): Linear, weak-field regime. (Right): Nonlinear, strong-field regime.



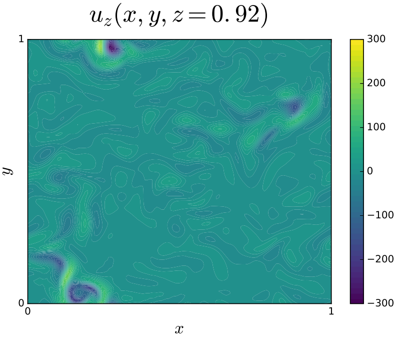


Figure: Pr $= 1$, $q = 1$, Ek $= 5 \times 10^{-6}$, Ra/Ra$_c = 0.98$.

Conclusions & Further Work

Conclusions:

- A transition to large-scale convection occurs when the magnetic field becomes sufficiently strong.
- The strong magnetic field allows the dynamo to sustain itself below the onset of convection, in the subcritical regime.

Further Work:

- Expand the explored parameter space, particularly decreasing Ekman number and moving further into the subcritical regime.
- Perform numerical simulations in a spherical dynamo model.