Subcritical Cartesian convection driven dynamos at low Ekman

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Planetary Dynamos

• Many planets have magnetic fields which are generated by dynamo action.

• In magnetohydrodynamics (MHD) convection in a rotating, electrically conducting fluid acts to maintain a magnetic field.

• This fluid can be driven by gradual cooling in the interior of the planet.

• This can happen if the convection is able to produce a magnetic field strong enough to alter the structure of the convective flows.

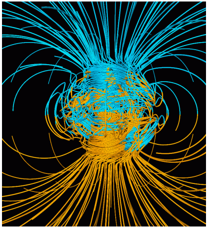


Figure: Glatzmaier & Roberts (1995)

The Martian Dynamo

• Some planets such as Mars do not presently have a magnetic field, but show evidence of having one in the past.

• Rocks on the surface show strong remnant magnetisation (Acuña et al., 1999).

• Studies observe that the cessation of the Martian dynamo occured rapidly (Lillis et al., 2008).

• One possible cause of this sudden termination is subcritical dynamo action.

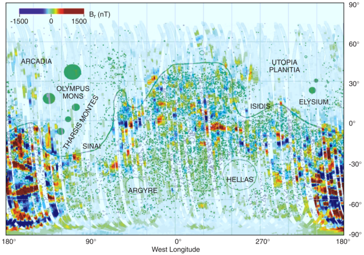
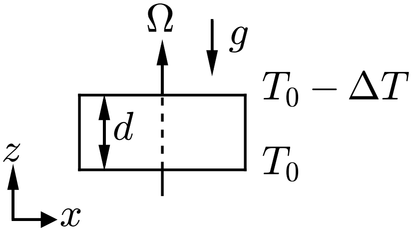


Figure: Acuña et al. (1999)

Model

- Aim to reproduce and extend the results of Stellmach & Hansen (2004) using the pseudospectral code of Cattaneo, Emonet & Weiss (2003).
- Convection-driven dynamo simulations in a rotating plane layer.
- Electrically conducting Boussinesq fluid.
- Constant rotation, $\mathbf{\Omega}$, aligned with gravity.
- Periodic boundaries in (x, y), stress-free, impermeable boundaries in z.
- Magnetic boundary conditions are electrically insulating.



Governing Equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla P + \mathrm{Pr}\nabla^2\boldsymbol{u} - \frac{\mathrm{Pr}}{\mathrm{Ek}}\hat{\boldsymbol{z}}\times\boldsymbol{u} + \boldsymbol{J}\times\boldsymbol{B} + \mathrm{Pr}\mathrm{Ra}\,T\hat{\boldsymbol{z}}, \quad (1)$$

$$\partial_t \boldsymbol{B} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} - \boldsymbol{B}\cdot\nabla\boldsymbol{u} = \frac{\mathrm{Pr}}{\mathrm{Pm}}\nabla^2\boldsymbol{B}, \quad (2)$$

$$\partial_t T + \boldsymbol{u}\cdot\nabla T = u_z + \nabla^2 T, \quad (3)$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad (4)$$

$$\nabla\cdot\boldsymbol{B} = 0. \quad (5)$$

Dimensionless parameters:

$$\mathrm{Pr} = \frac{\nu}{\kappa}, \quad \mathrm{Pm} = \frac{\nu}{\eta}, \quad \mathrm{Ra} = \frac{g\alpha\Delta T d^3}{\kappa\nu}, \quad \mathrm{Ek} = \frac{\nu}{2\Omega d^2}. \quad (6)$$

Stellmach & Hansen (2004)

- Convection-driven dynamo and non-magnetic convection simulations in a rotating plane layer.
- Compare flow structures finding a transition from small-scale motions relatively unaffected by the magnetic field to large-scale motions controlled by Lorentz forces.
- Show that in the nonlinear regime the magnetic field promotes convection, increasing heat transport and flow amplitude.
- Manage to **sustain a subcritical dynamo** at a single Ekman number.

Moderately Supercritical Dynamos

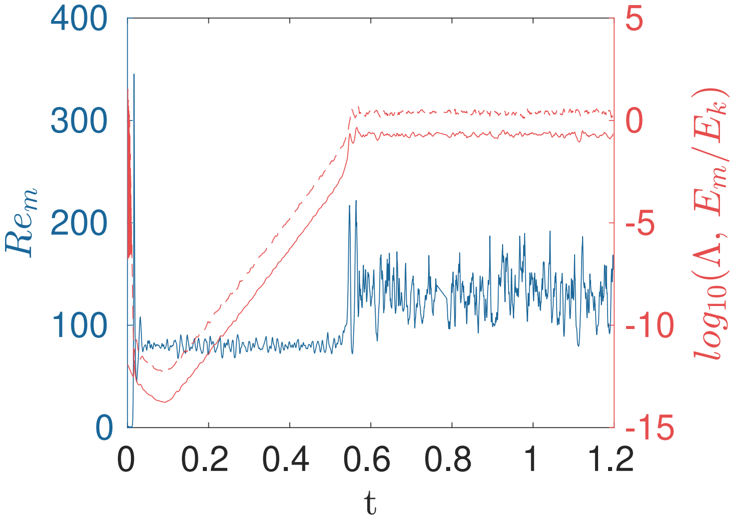


Figure: $\mathrm{Pr} = 1$, $\mathrm{Pm} = 1$, $\mathrm{Ek} = 5 \times 10^{-6}$, $\mathrm{Ra}/\mathrm{Ra}_c = 1.18$.

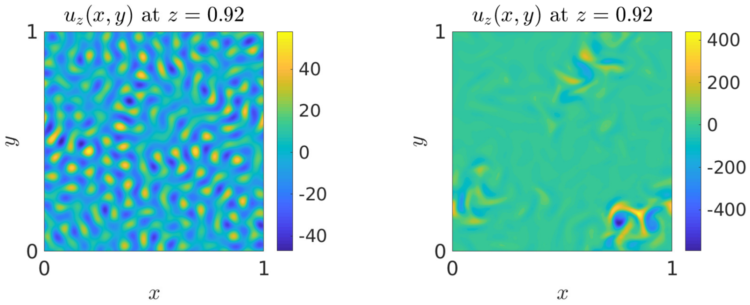


Figure: (Left): Kinematic regime. (Right): Nonlinear regime.

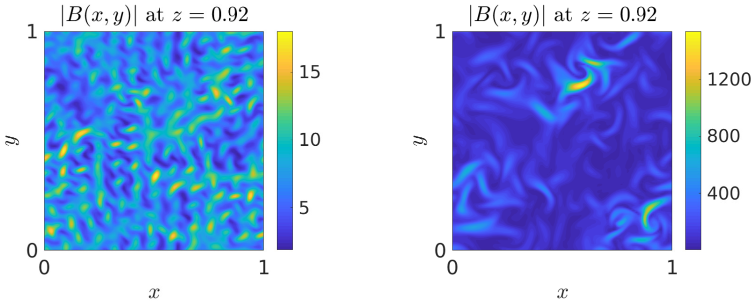


Figure: (Left): Kinematic regime. (Right): Nonlinear regime.

Kinematic vs Nonlinear Spectra

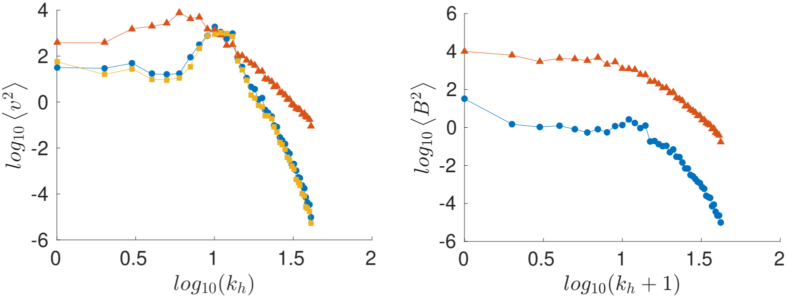


Figure: (Left): KE spectra and (Right): Magnetic spectra for the nonlinear regime (red), kinematic regime (blue) and for nonmagnetic rotating convection (amber).

Subcritical Dynamos

• Subcritical dynamo action is dynamo action for convective forcing below the threshold necessary for convective motions to occur in the absence of magnetic fields.
• The energy required to sustain the dynamo is far less than required to initiate the dynamo in the absence of the strong magnetic field.

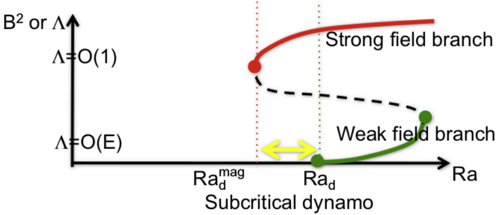


Figure: Hori & Wicht (2013)

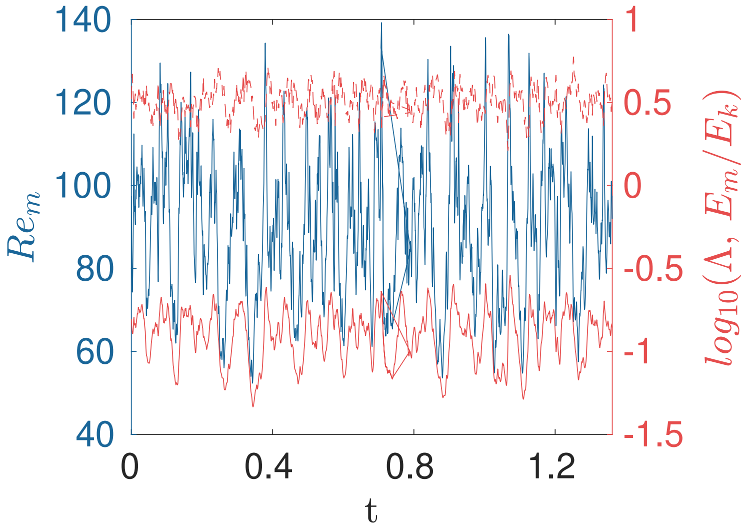


Figure: Pr $= 1$, Pm $= 1$, Ek $= 5 \times 10^{-6}$, Ra$/$Ra$_c = 0.93$.

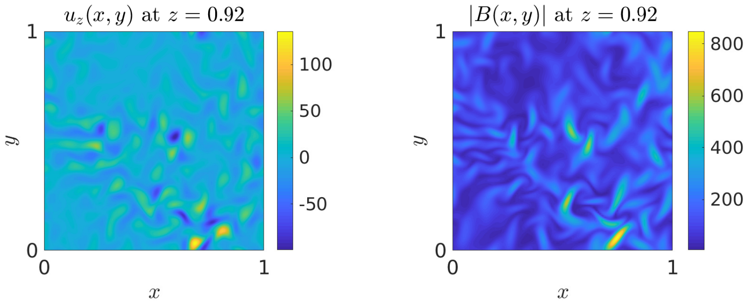


Figure: (Left): $u_z(x, y)$ and (Right): $|B(x, y)|$.

Subcritical Spectra

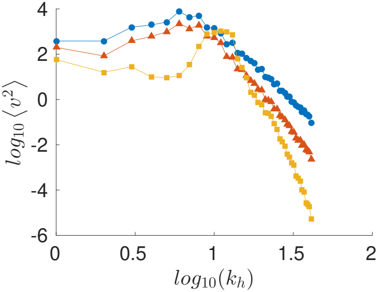


Figure: (Left): KE spectra and (Right): Magnetic spectra for supercritical Rayleigh number (blue), subcritical Rayleigh number (red) and for supercritical nonmagnetic rotating convection (amber).

Parameter Space so far

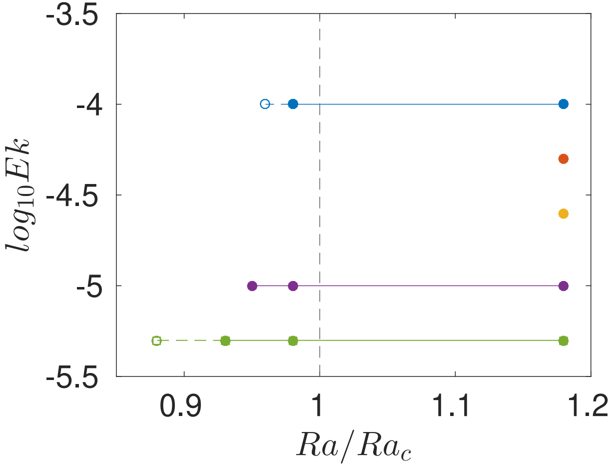


Figure: (blue): highest Ekman (slowest rotation), (green): lowest Ekman (fastest rotation).

Magnetic Reynolds number

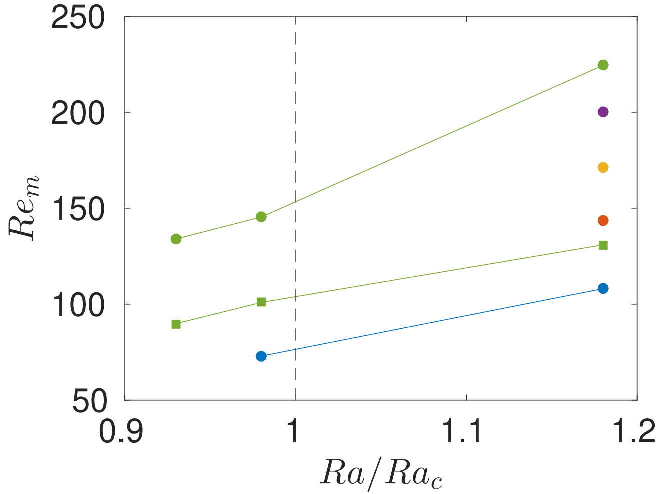


Figure: (blue): highest Ekman (slowest rotation), (green): lowest Ekman (fastest rotation).

Elsasser number

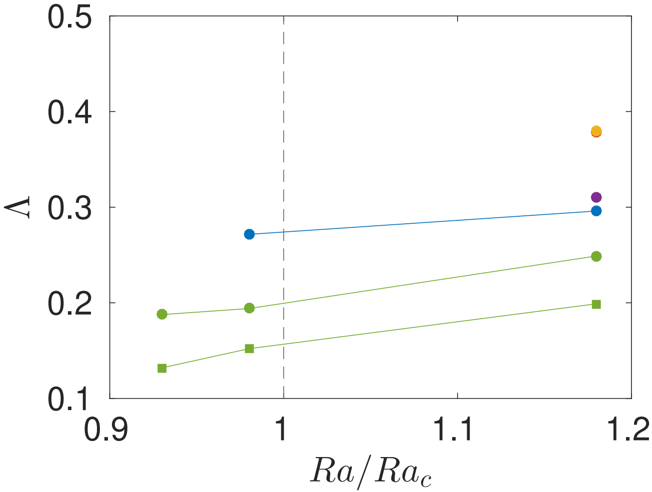


Figure: (blue): highest Ekman (slowest rotation), (green): lowest Ekman (fastest rotation).

Magnetic to Kinetic Energy ratio

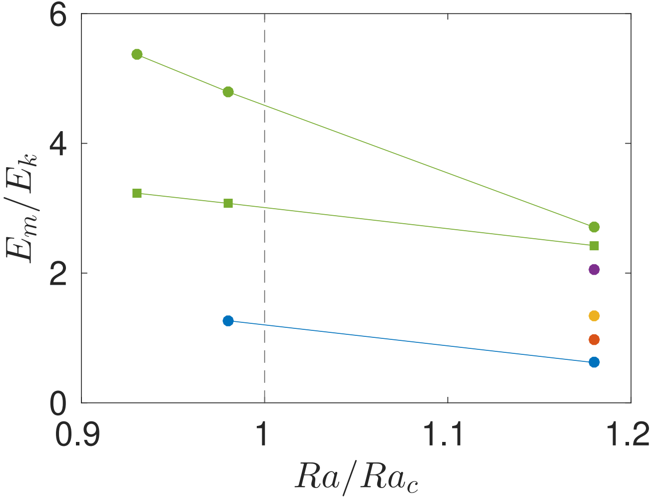


Figure: (blue): highest Ekman (slowest rotation), (green): lowest Ekman (fastest rotation).

Conclusions:

- A transition to large-scale convection occurs when the magnetic field becomes sufficiently strong.
- The strong magnetic field allows the dynamo to sustain itself below the onset of convection, in the subcritical regime.
- More rapid rotation may lead to dynamo action deeper into the subcritical regime.

Further Work:

- Expand the explored parameter space, particularly decreasing Ekman number and moving further into the subcritical regime.
- Perform numerical simulations in a spherical dynamo model.