Subcritical Cartesian convection driven dynamos at low Ekman number

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**MHD Days and
GdRI Dynamo Meeting**

Planetary Dynamos

- Many planets have magnetic fields which are generated by dynamo action.
- In magnetohydrodynamics (MHD) convection in a rotating, electrically conducting fluid acts to maintain a magnetic field.
- This fluid can be driven by gradual cooling in the interior of the planet.
- This can happen if the convection is able to produce a magnetic field strong enough to alter the structure of the convective flows.

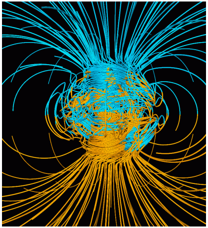


Figure: Glatzmaier & Roberts (1995)

The Martian Dynamo

- Some planets such as Mars do not presently have a magnetic field, but show evidence of having one in the past.
- Rocks on the surface show strong remnant magnetisation (Acuña et al., 1999).
- Studies observe that the cessation of the Martian dynamo occured rapidly (Lillis et al., 2008).
- One possible cause of this sudden termination is subcritical dynamo action.

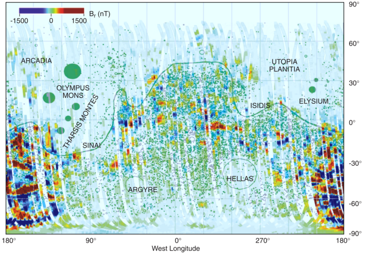


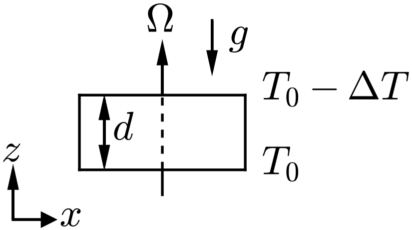
Figure: Acuña et al. (1999)

Stellmach & Hansen (2004)

- Convection-driven dynamo and non-magnetic convection simulations in a rotating plane layer.
- Compare flow structures finding a transition from small-scale motions relatively unaffected by the magnetic field to large-scale motions controlled by Lorentz forces.
- Show that in the nonlinear regime the magnetic field promotes convection, increasing heat transport and flow amplitude.
- Manage to **sustain a subcritical dynamo** at a single Ekman number.

Model

- Aim to reproduce and extend the results of Stellmach & Hansen (2004) using the pseudospectral code of Cattaneo, Emonet & Weiss (2003).
- Convection-driven dynamo simulations in a rotating plane layer.
- Electrically conducting Boussinesq fluid.
- Constant rotation, $\mathbf{\Omega}$, aligned with gravity.
- Periodic boundaries in (x, y), stress-free, impermeable boundaries in z.
- Magnetic boundary conditions are electrically insulating.



Governing Equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla P + \mathrm{Pr}\nabla^2\boldsymbol{u} - \frac{\mathrm{Pr}}{\mathrm{Ek}}\hat{\boldsymbol{z}}\times\boldsymbol{u} + \boldsymbol{J}\times\boldsymbol{B} + \mathrm{Pr}\mathrm{Ra}\,T\hat{\boldsymbol{z}}, \quad (1)$$

$$\partial_t \boldsymbol{B} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} - \boldsymbol{B}\cdot\nabla\boldsymbol{u} = \frac{\mathrm{Pr}}{\mathrm{Pm}}\nabla^2\boldsymbol{B}, \quad (2)$$

$$\partial_t T + \boldsymbol{u}\cdot\nabla T = u_z + \nabla^2 T, \quad (3)$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad (4)$$

$$\nabla\cdot\boldsymbol{B} = 0. \quad (5)$$

Dimensionless parameters:

$$\mathrm{Pr} = \frac{\nu}{\kappa}, \quad \mathrm{Pm} = \frac{\nu}{\eta}, \quad \mathrm{Ra} = \frac{g\alpha\Delta T d^3}{\kappa\nu}, \quad \mathrm{Ek} = \frac{\nu}{2\Omega d^2}. \quad (6)$$

Subcritical Dynamos

- Subcritical dynamo action is dynamo action for convective forcing below the threshold necessary for convective motions to occur in the absence of magnetic fields.
- The energy required to sustain the dynamo is far less than required to initiate the dynamo in the absence of the strong magnetic field.

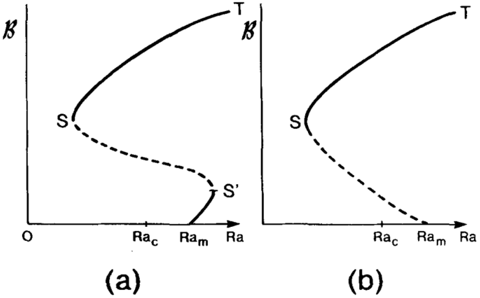


Figure: Roberts (1978, 1979)

Moderately Supercritical Dynamos

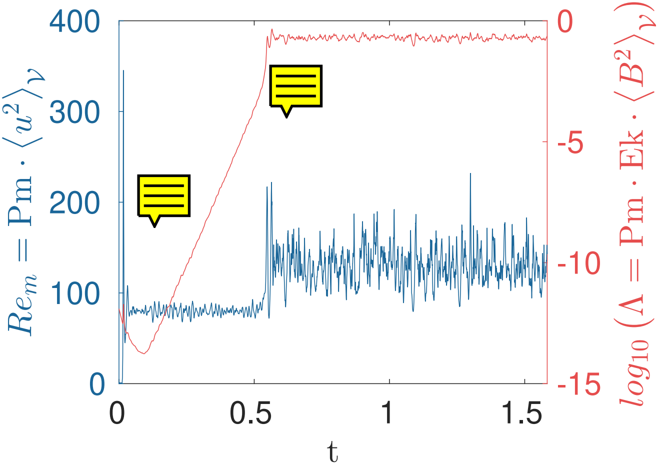
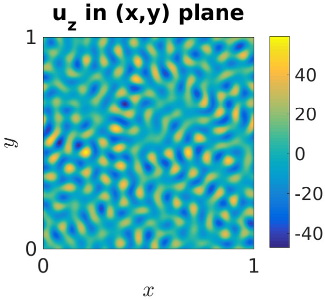
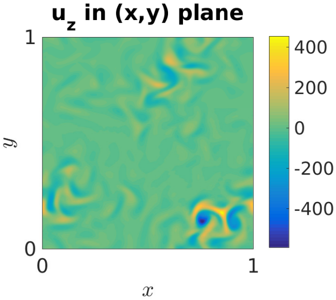


Figure: $\mathrm{Pr} = 1$, $\mathrm{Pm} = 1$, $\mathrm{Ek} = 5 \times 10^{-6}$, $\mathrm{Ra}/\mathrm{Ra}_c = 1.18$.

Kinematic (left) vs Nonlinear (right) Flows





Kinematic (left) vs Nonlinear (right) Flows

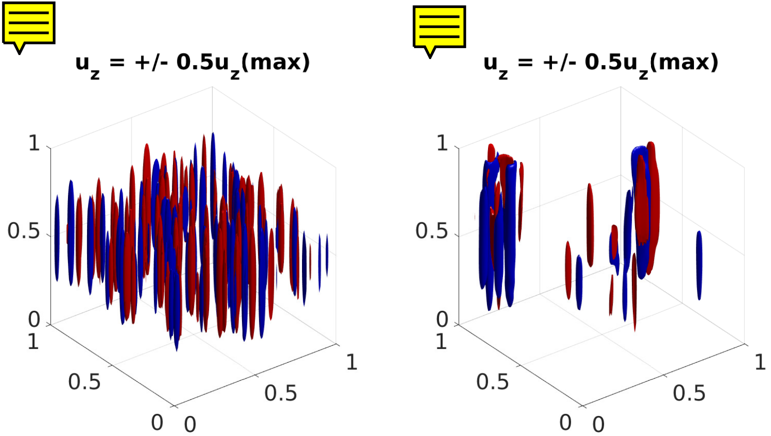
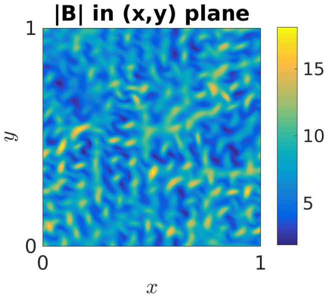
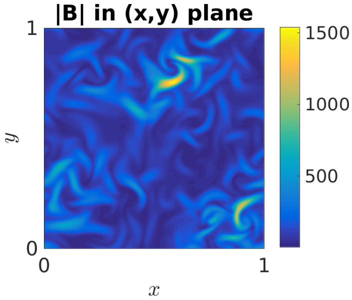


Figure: Isosurfaces of the axial velocity, u_z.

Kinematic (left) vs Nonlinear (right) Magnetic Fields





Kinematic (left) vs Nonlinear (right) Mean Fields

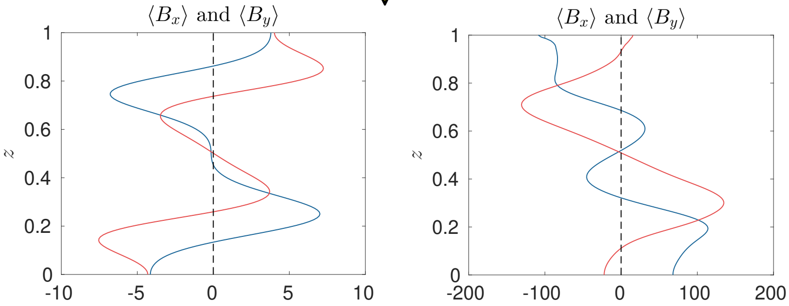


Figure: (Blue): B_x and (Red): B_y averaged over (x, y).

Kinematic vs Nonlinear Spectra

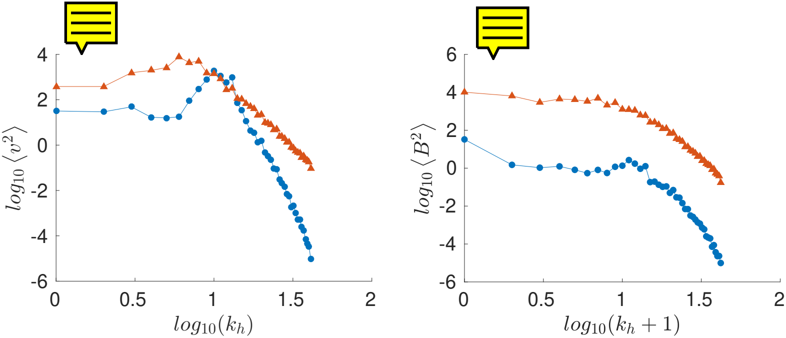


Figure: (Left): KE spectra and (Right): Magnetic spectra for the kinematic regime (blue) and nonlinear regime (red). $k_h = \sqrt{k_x^2 + k_y^2}$ is the horizontal wavenumber.

Subcritical Dynamos

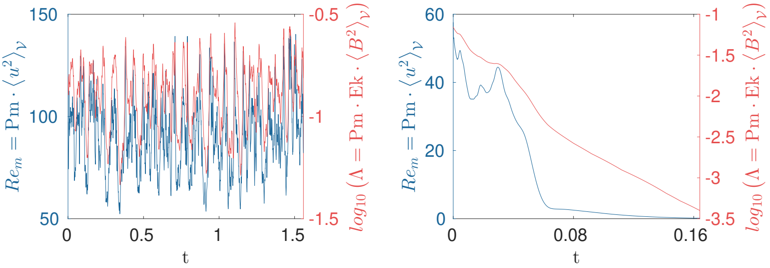


Figure: $\mathrm{Pr} = 1$, $\mathrm{Pm} = 1$, $\mathrm{Ek} = 5 \times 10^{-6}$, $\mathrm{Ra}/\mathrm{Ra}_c = 0.93$ (left) and $\mathrm{Ra}/\mathrm{Ra}_c = 0.88$ (right).

Parameter Space so far

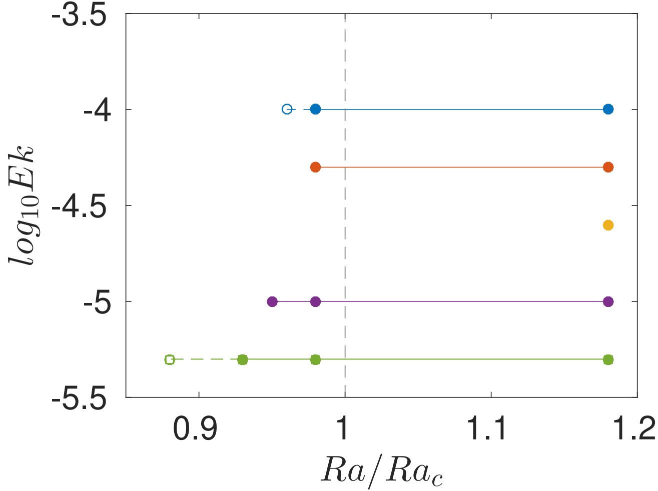


Figure: (blue): highest Ekman (slowest rotation), (green): lowest Ekman (fastest rotation).

Conclusions & Further Work

Conclusions:

- A transition to large-scale convection occurs when the magnetic field becomes sufficiently strong.
- The strong magnetic field allows the dynamo to sustain itself below the onset of convection, in the subcritical regime.
- More rapid rotation may lead to dynamo action deeper into the subcritical regime.

Further Work:

- Expand the explored parameter space, particularly decreasing Ekman number and moving further into the subcritical regime.
- Perform numerical simulations in a spherical dynamo model.