Subcritical Cartesian convection driven dynamos at low Ekman number

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MHD Days and GdRI Dynamo Meeting
Planetary Dynamos

- Many planets have magnetic fields which are generated by dynamo action.
- In magnetohydrodynamics (MHD) convection in a rotating, electrically conducting fluid acts to maintain a magnetic field.
- This fluid can be driven by gradual cooling in the interior of the planet.
- This can happen if the convection is able to produce a magnetic field strong enough to alter the structure of the convective flows.

Figure: Glatzmaier & Roberts (1995)
The Martian Dynamo

- Some planets such as Mars do not presently have a magnetic field, but show evidence of having one in the past.
- Rocks on the surface show strong remnant magnetisation (Acuña et al., 1999).
- Studies observe that the cessation of the Martian dynamo occurred rapidly (Lillis et al., 2008).
- One possible cause of this sudden termination is subcritical dynamo action.

Figure: Acuña et al. (1999)
• Convection-driven dynamo and non-magnetic convection simulations in a rotating plane layer.
• Compare flow structures finding a transition from small-scale motions relatively unaffected by the magnetic field to large-scale motions controlled by Lorentz forces.
• Show that in the nonlinear regime the magnetic field promotes convection, increasing heat transport and flow amplitude.
• Manage to **sustain a subcritical dynamo** at a single Ekman number.

Convection-driven dynamo simulations in a rotating plane layer.

Electrically conducting Boussinesq fluid.

Constant rotation, \( \Omega \), aligned with gravity.

Periodic boundaries in \((x, y)\), stress-free, impermeable boundaries in \(z\).

Magnetic boundary conditions are electrically insulating.
Governing Equations

\[ \partial_t u + u \cdot \nabla u = -\nabla P + \Pr \nabla^2 u - \frac{\Pr}{\text{Ek}} \hat{z} \times u + J \times B + \Pr \text{Ra} \, T \hat{z}, \quad (1) \]

\[ \partial_t B + u \cdot \nabla B - B \cdot \nabla u = \frac{\Pr}{\text{Pm}} \nabla^2 B, \quad (2) \]

\[ \partial_t T + u \cdot \nabla T = u_z + \nabla^2 T, \quad (3) \]

\[ \nabla \cdot u = 0, \quad (4) \]

\[ \nabla \cdot B = 0. \quad (5) \]

Dimensionless parameters:

\[ \Pr = \frac{\nu}{\kappa}, \quad \text{Pm} = \frac{\nu}{\eta}, \quad \text{Ra} = \frac{g \alpha \Delta T d^3}{\kappa \nu}, \quad \text{Ek} = \frac{\nu}{2 \Omega d^2}. \quad (6) \]
Subcritical Dynamos

- Subcritical dynamo action is dynamo action for convective forcing below the threshold necessary for convective motions to occur in the absence of magnetic fields.
- The energy required to sustain the dynamo is far less than required to initiate the dynamo in the absence of the strong magnetic field.

Figure: Roberts (1978, 1979)
Figure: \( \text{Pr} = 1, \text{Pm} = 1, \text{Ek} = 5 \times 10^{-6}, \frac{\text{Ra}}{\text{Ra}_c} = 1.18. \)
Kinematic (left) vs Nonlinear (right) Flows

$u_z$ in $(x,y)$ plane

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Kinematic (left) vs Nonlinear (right) Flows

$u_z = +/- 0.5 u_z(\text{max})$

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Figure: Isosurfaces of the axial velocity, $u_z$. 
Kinematic (left) vs Nonlinear (right) Magnetic Fields

$|B|$ in $(x,y)$ plane

$|B|$ in $(x,y)$ plane
Kinematic (left) vs Nonlinear (right) Mean Fields

Figure: (Blue): $B_x$ and (Red): $B_y$ averaged over $(x, y)$. 
Kinematic vs Nonlinear Spectra

Figure: (Left): KE spectra and (Right): Magnetic spectra for the kinematic regime (blue) and nonlinear regime (red). $k_h = \sqrt{k_x^2 + k_y^2}$ is the horizontal wavenumber.
Subcritical Dynamos

Figure: Pr = 1, Pm = 1, Ek = $5 \times 10^{-6}$, $Ra/Ra_c = 0.93$ (left) and $Ra/Ra_c = 0.88$ (right).
Parameter Space so far

Figure: (blue): highest Ekman (slowest rotation), (green): lowest Ekman (fastest rotation).
Conclusions:
• A transition to large-scale convection occurs when the magnetic field becomes sufficiently strong.
• The strong magnetic field allows the dynamo to sustain itself below the onset of convection, in the subcritical regime.
• More rapid rotation may lead to dynamo action deeper into the subcritical regime.

Further Work:
• Expand the explored parameter space, particularly decreasing Ekman number and moving further into the subcritical regime.
• Perform numerical simulations in a spherical dynamo model.