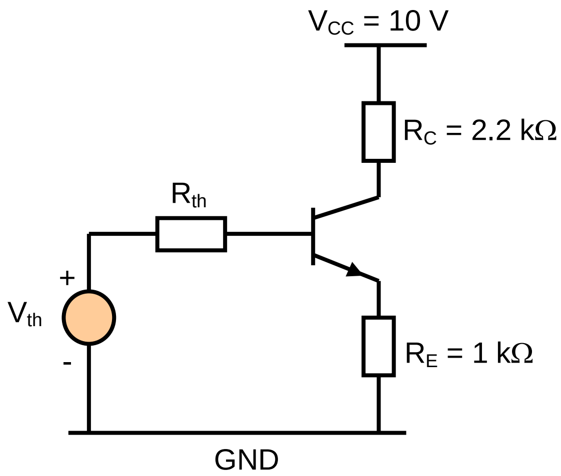
Tutorial 3 – Answers

Question 1

We use Thévenin's theorem to simplify the DC analysis. For the DC mode, the amplifier circuit is thus equivalent to the circuit depicted below, with $V_{th} = \frac{R_2 V_{CC}}{R_1 + R_2} \approx 2.7$ V and $R_{th} = \frac{R_1 R_2}{R_1 + R_2} \approx 7.3$ kΩ.



We assume that the BJT is in the forward active mode (to be checked later). Under this assumption, we show that:

$$V_{th} \approx R_{th} I_{B0} + 0.7 + R_E I_{E0} = R_{th} I_{B0} + 0.7 + R_E (1 + \beta_F) I_{B0} \approx [R_{th} + R_E \beta_F] I_{B0} + 0.7$$

⇨ $I_{B0} \approx \frac{V_{th} - 0.7}{R_{th} + R_E \beta_F} \approx 18.6$ μA.

⇨ $I_{C0} = \beta_F I_{B0} \approx 1.86$ mA and $I_{E0} = (1 + \beta_F) I_{B0} \approx I_{C0} \approx 1.86$ mA.

Note that $I_{C0} \approx I_{E0} \approx \beta_F \frac{V_{th} - 0.7}{R_{th} + R_E \beta_F}$. This equation indicates that the value of β_F has very little effect on the DC operating point.

The excellent stability of the DC operating point is actually due to the use of negative feedback. In fact, without feedback ($R_E = 0$), we would obtain

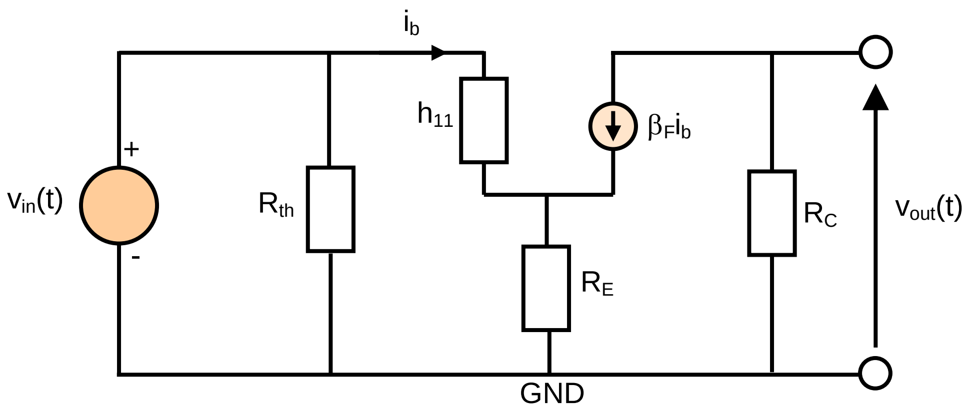
$$I_{C0} \approx I_{E0} \approx \beta_F \frac{V_{th} - 0.7}{R_{th}},$$

thus implying that the DC operating point would be very much dependent on the value of β_F.

We also have $V_{CC} = R_C I_{C0} + V_{CE0} + R_E I_{E0} \approx (R_C + R_E) I_{C0} + V_{CE0}$

⇨ $V_{CE0} \approx V_{CC} - (R_C + R_E) I_{C0} \approx$ 4.08 V. The transistor is thus in the forward active mode.

As for the AC analysis, the small-signal model of our feedback amplifier is depicted below.



We can show that $v_{out}(t) = -R_C \beta_F i_b$.

In addition, we can write $v_{in}(t) = h_{11} i_b + R_E (1 + \beta_F) i_b \approx (h_{11} + R_E \beta_F) i_b$.

The small-signal voltage gain of the feedback amplifier is thus given by

$$A_v = \frac{v_{out}(t)}{v_{in}(t)} \approx -\frac{\beta_F R_C}{h_{11} + R_E \beta_F}.$$

At room temperature, we can write $h_{11} \approx \frac{25\ \text{mV}}{I_{B0}}$, with $I_{B0} \approx 18.6\ \mu$A. So, we have $h_{11} \approx 1.34$ kΩ.

Since $h_{11} << R_E \beta_F$, we finally have

$$A_v \approx -\frac{R_C}{R_E} \approx -2.2.$$

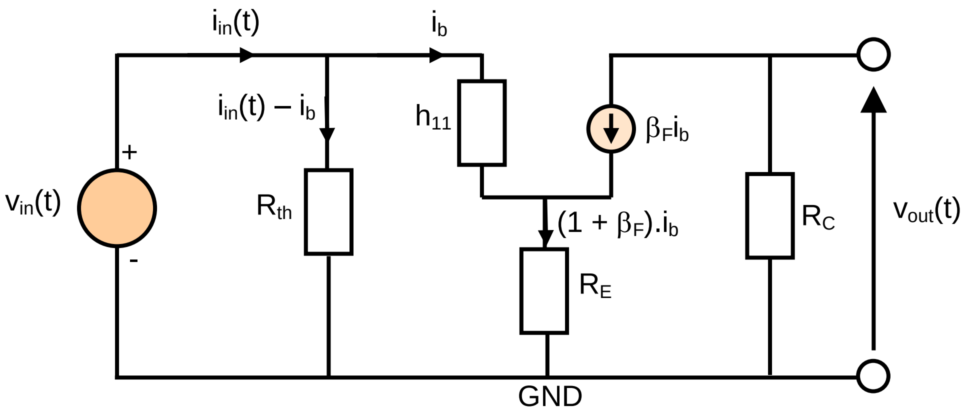
Without feedback (R_E = 0), we would obtain

$$A_v \approx -\beta_F \frac{R_C}{h_{11}},$$

thus resulting in a much higher small-signal voltage gain equal to -163. Negative feedback does significantly reduce the gain of an amplifier, but the good thing is that this gain does not depend on the value of β_F and is therefore very stable.

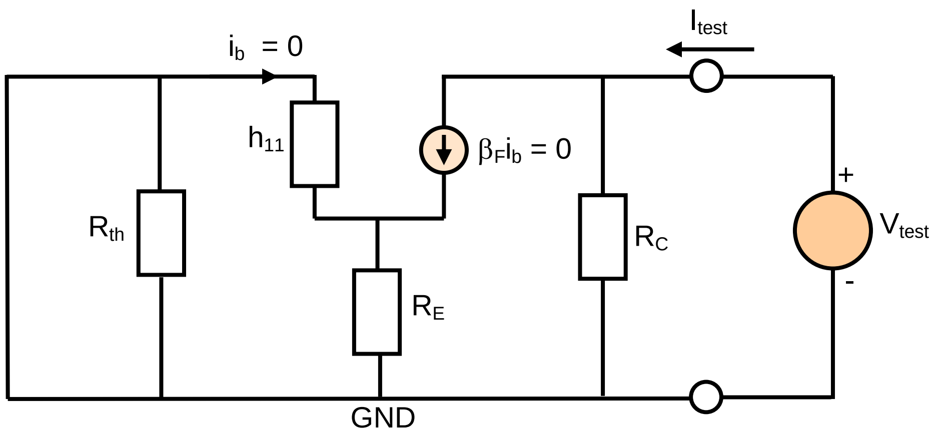
The small-signal input resistance is given by

$$r_{in} = \frac{v_{in}(t)}{i_{in}(t)} = \frac{[h_{11} + R_E (1+\beta_F)]R_{th}}{h_{11} + R_E (1+\beta_F) + R_{th}} \approx \frac{R_E \beta_F R_{th}}{R_E \beta_F + R_{th}} \approx 6.8 \text{ k}\Omega.$$



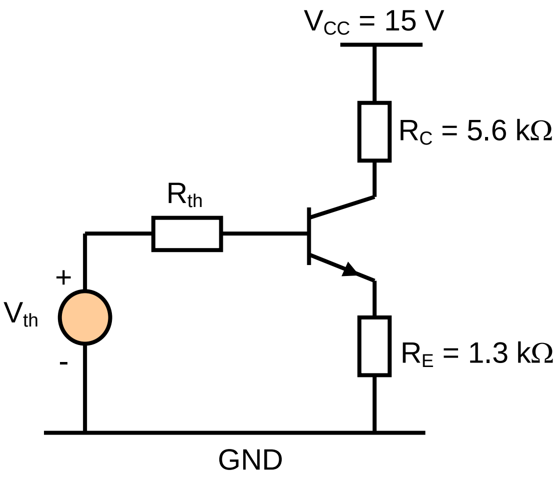
Without feedback (R_E = 0), we would obtain $r_{in} = \frac{h_{11} R_{th}}{h_{11} + R_{th}} \approx$ 1.1 kΩ. So, negative feedback has increased the input resistance, which is really good!

The small-signal output resistance is given by $r_{out} = R_C$ = 2.2 kΩ (same result as without feedback).



Question 2

We use Thévenin's theorem to simplify the DC analysis. For the DC mode, the amplifier circuit is thus equivalent to the circuit depicted below, with $V_{th} = \frac{R_2 V_{CC}}{R_1 + R_2}$ ≈ 2.05 V and $R_{th} = \frac{R_1 R_2}{R_1 + R_2}$ ≈ 11.2 kΩ.



Now, we assume that the BJT is in the forward active mode (to be checked later). Under this assumption, we show that:

$$V_{th} \approx R_{th} I_{B0} + 0.7 + R_E I_{E0} = R_{th} I_{B0} + 0.7 + R_E (1 + \beta_F) I_{B0} \approx (R_{th} + R_E \beta_F) I_{B0} + 0.7$$

⇨ $I_{B0} \approx \frac{V_{th} - 0.7}{R_{th} + R_E \beta_F}$ ≈ 9.6 μA.

⇨ $I_{C0} = \beta_F I_{B0}$ ≈ 0.96 mA and $I_{E0} = (1 + \beta_F) I_{B0} \approx I_{C0}$ = 0.96 mA.

Note that $I_{C0} \approx I_{E0} \approx \beta_F \frac{V_{th} - 0.7}{R_{th} + R_E \beta_F}$. This equation indicates that the value of β_F has very little effect on the DC operating point.

The excellent stability of the DC operating point is actually due to the use of negative feedback. In fact, without feedback (R_E = 0), we would obtain

$$I_{C0} \approx I_{E0} \approx \beta_F \frac{V_{th} - 0.7}{R_{th}},$$

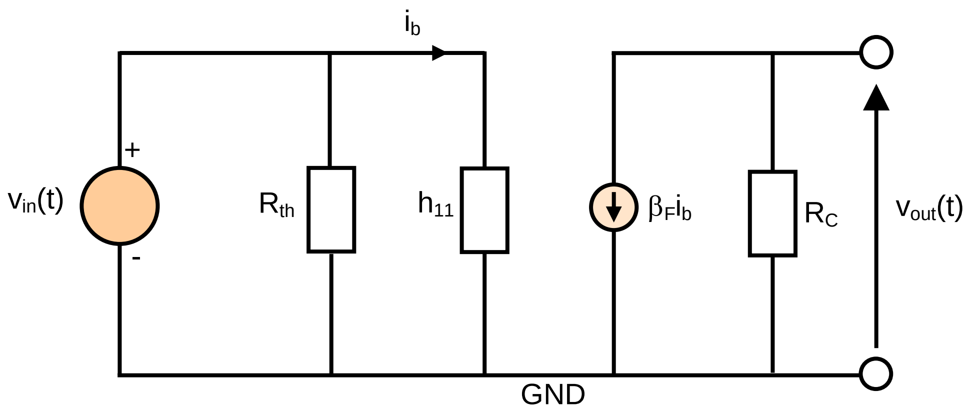
thus implying that the DC operating point would be very much dependent on the value of β_F .

We also have $V_{CC} = R_C I_{C0} + V_{CE0} + R_E I_{E0} \approx (R_C + R_E) I_{C0} + V_{CE0}$

⇨ $V_{CE0} \approx V_{CC} - (R_C + R_E) I_{C0} \approx$ 8.4 V. The transistor is thus in the forward active mode.

As for the AC analysis, the small-signal model of our amplifier is depicted below. We have assumed that the decoupling capacitance is equivalent to a short circuit at the frequency of the input signal.

We thus obtain the small-signal model of a classical common-emitter amplifier similar to that described in the lecture notes.



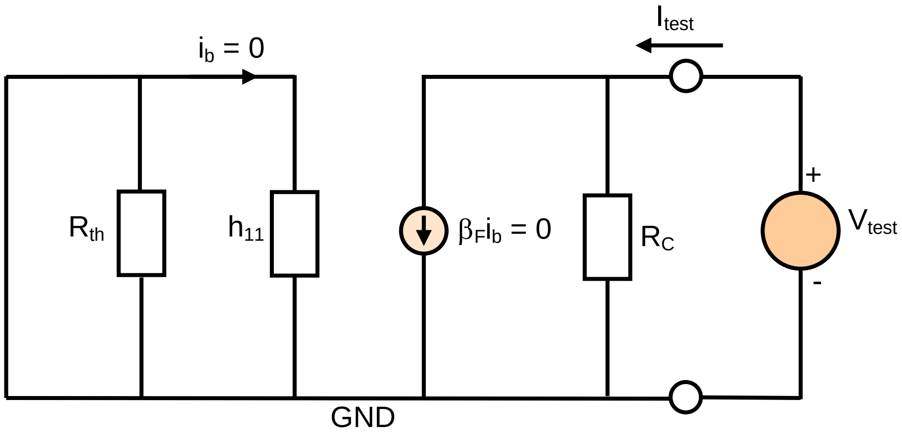
We can show that $v_{out}(t) = -R_C \beta_F i_b$. In addition, we can write $v_{in}(t) = h_{11} i_b$. The small-signal voltage gain of the common-emitter amplifier is thus given by

$$A_v = \frac{v_{out}(t)}{v_{in}(t)} = -\beta_F \frac{R_C}{h_{11}} .$$

At room temperature, we have $h_{11} \approx \frac{25 \text{ mV}}{I_{B0}}$, with $I_{B0} \approx 9.6$ µA. So, we have $h_{11} \approx 2.6$ kΩ. We can finally write $A_v = -\beta_F \frac{R_C}{h_{11}} \approx$ -215, which is a much higher voltage gain than that achieved in

Question 1, thanks to the decoupling capacitance. Note that this gain depends on the value of β_F and is therefore unstable.

The small-signal input resistance is given by $r_{in} = \frac{h_{11} R_{th}}{h_{11} + R_{th}} \approx$ 2.1 kΩ, whereas the small-signal output resistance is given by $r_{out} = R_C$ = 5.6 kΩ.



Question 3

The small-signal voltage gain of the cascaded amplifier is given by

$$A_{v,c} = (A_v)^2 \frac{r_{in}}{r_{in} + R_S} \frac{r_{in}}{r_{in} + r_{out}} \frac{R_L}{R_L + r_{out}},$$

with

- $A_v = -\beta_F \frac{R_C}{h_{11}} \approx$ -215;
- $r_{in} = \frac{h_{11} R_{th}}{h_{11} + R_{th}} \approx$ 2.1 kΩ;
- $r_{out} = R_C$ = 5.6 kΩ.

Hence, we finally have:

$$A_{v,c} \approx (-215)^2 \times \frac{2.1}{2.1+1} \times \frac{2.1}{2.1+5.6} \times \frac{10}{10+5.6} \approx 46{,}225 \times 0.677 \times 0.273 \times 0.641 \approx 5476.$$

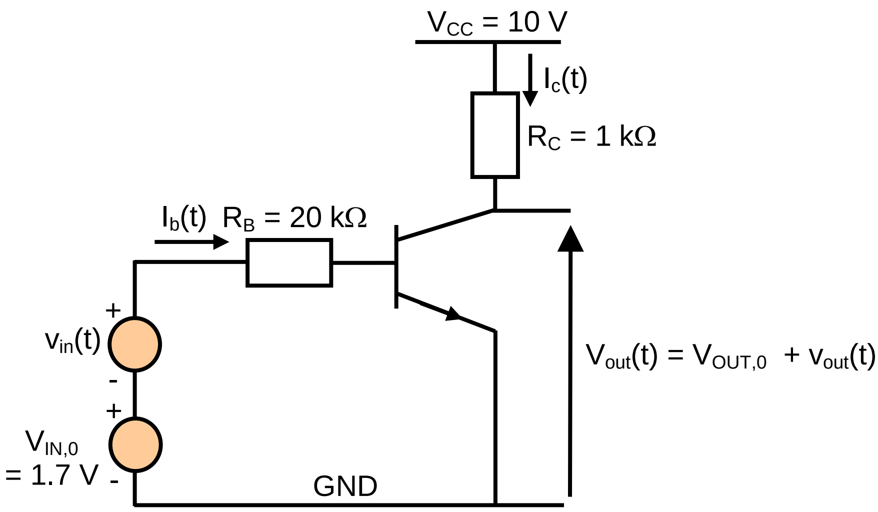
If the load resistance R_L was equal to 10 Ω instead of 10 kΩ, the small-signal voltage gain would be

$$A_{v,c} \approx (-215)^2 \times \frac{2.1}{2.1+1} \times \frac{2.1}{2.1+5.6} \times \frac{0.01}{0.01+5.6} \approx 46{,}225 \times 0.677 \times 0.273 \times 0.00178 \approx 15.2.$$

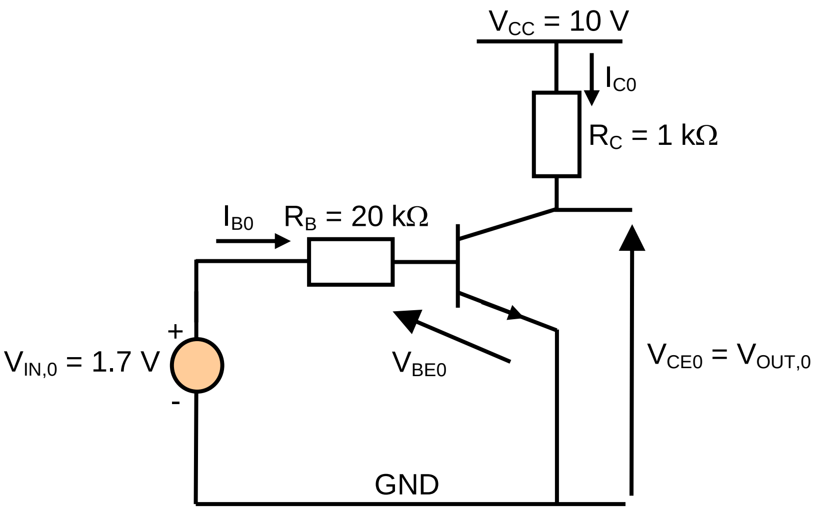
The voltage gain of the cascaded amplifier would thus be significantly reduced. We conclude that the overall small-signal voltage gain $A_{V,C}$ strongly depends on the input resistance R_L of the loading circuit. This is due to the rather high output resistance r_{out} = 5.6 kΩ of this amplifier.

Question 4

We show below the structure of the amplifier designed in Question 2 of Tutorial 2.



Let us first perform the DC analysis of this amplifier using the circuit depicted below.

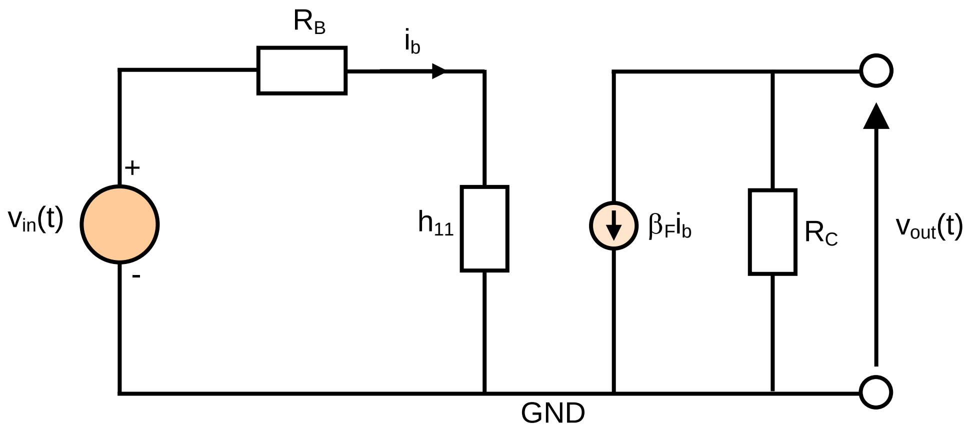


As usual, we assume that the BJT is in the forward active mode (to be checked later). We can write: $V_{IN,0} \approx R_B I_{B0} + 0.7$ => $I_{B0} \approx \frac{V_{IN,0} - 0.7}{R_B} \approx 50\ \mu A$.

The collector current is thus given by $I_{C0} = \beta_F I_{B0} \approx \beta_F \frac{V_{IN,0} - 0.7}{R_B} \approx 5$ mA, whereas the voltage V_{CE} is obtained using $V_{CC} = R_C I_{C0} + V_{CE0}$ => $V_{CE0} = V_{CC} - R_C I_{C0} \approx V_{CC} - \beta_F \frac{R_C}{R_B}\left(V_{IN,0} - 0.7\right) \approx 5$ volts.

This result indicates that the BJT is in the forward active mode and is in fact ideally biased midway between V_{CC} = 10 volts and $V_{CE,sat} \approx 0.2$ volt, thus allowing for maximal symmetrical output voltage swing around the DC operating point (remember that $V_{CE0} = V_{OUT,0}$, here).

As for the AC analysis, the small-signal model of our amplifier is depicted below.

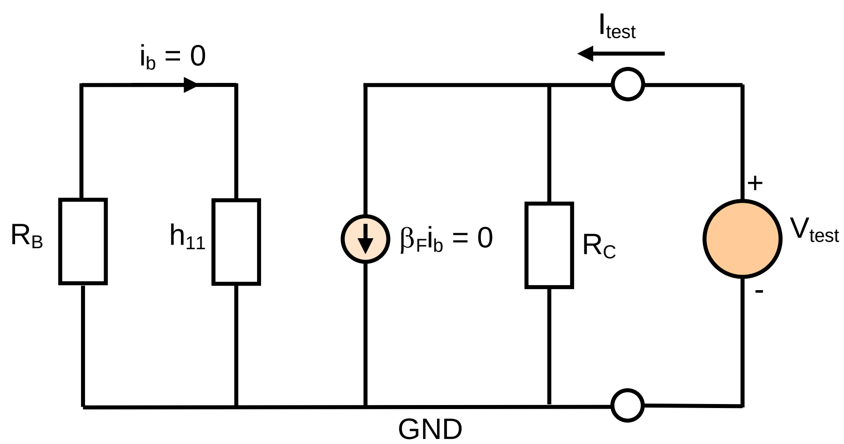


We can show that $v_{out}(t) = -R_C\,\beta_F\,i_b$. In addition, we can write $v_{in}(t) = (R_B + h_{11})i_b$. The small-signal voltage gain of the common-emitter amplifier is thus given by

$$A_V = \frac{v_{out}(t)}{v_{in}(t)} = -\beta_F \frac{R_C}{R_B + h_{11}}.$$

At room temperature, we have $h_{11} \approx \frac{25\text{ mV}}{I_{B0}}$, with $I_{B0} \approx 50$ µA. So, we have $h_{11} \approx 500\ \Omega << R_B = 20$ kΩ. We can thus write $A_V \approx -\beta_F \frac{R_C}{R_B}$ ≈ -5, which is identical to the voltage gain obtained in Question 2 of Tutorial 2.

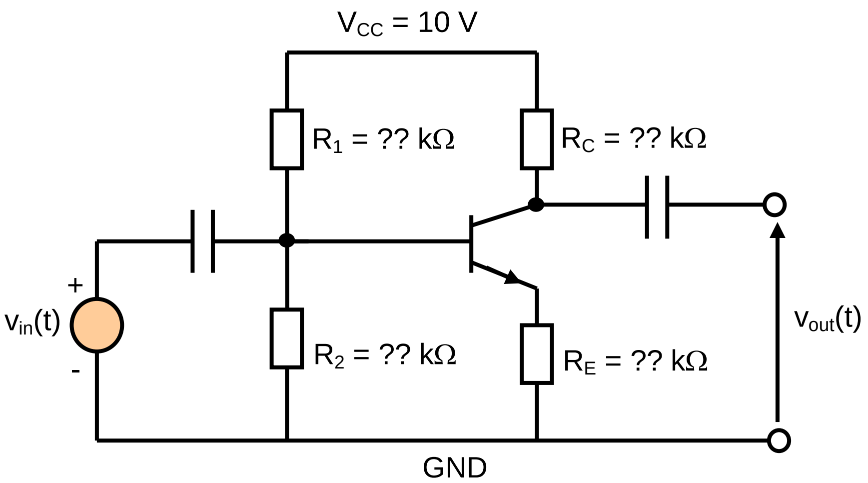
The small-signal input resistance is given by $r_{in} = h_{11} + R_B \approx R_B$ = 20 kΩ, whereas the small-signal output resistance is given by $r_{out} = R_C$ = 5.6 kΩ.



We can conclude that the results obtained with both DC and AC analysis are coherent with those derived in Question 2 of Tutorial 2 (fortunately!).

Question 5

The structure of our amplifier is depicted below.



The design issue consists of determining the values of the four resistances R_1, R_2, R_C and R_E so as to meet the specifications. To this end, we must remember several results that have previously been obtained for the common-emitter amplifier with negative feedback.

We start with the expression of the voltage gain:

$$A_v = \frac{v_{out}(t)}{v_{in}(t)} \approx -\frac{\beta_F R_C}{h_{11} + R_E \beta_F},$$

which can simplified as $A_v \approx -\frac{R_C}{R_E}$ as long as $h_{11} << R_E \beta_F$.

Assume that we indeed have $h_{11} << R_E \beta_F$ (to be checked later). In this case, a voltage gain of approximately -9 can be achieved by choosing, for example, R_C = 10 kΩ and R_E = 1 kΩ (thus leading to $A_v \approx$ -10, which gives us a bit of allowance).

Note that the output resistance r_{out} of the amplifier is given by $r_{out} = R_C = 10\ k\Omega$. This value tends to be a bit high as an output resistance but can still considered as acceptable. Obviously, we could reduce both R_C and R_E in order to decrease the output resistance while keeping the voltage gain unchanged. However, doing so would negatively impact some other parameters in this circuit. For instance, if R_E becomes too low, the condition $h_{11} << R_E \beta_F$ may not hold anymore. Ultimately, there are several conflicting constraints to consider when designing such a circuit, and a trade-off must be found.

Now, we are also required to maximise the output voltage swing. To this end, we must first find "where the cut-off mode starts" and "where the saturation mode starts".

Let us use the following notations for the compound signals (AC + DC) in the circuit: $I_c(t) = i_c(t) + I_{C0}$, $V_{ce}(t) = v_{ce}(t) + V_{CE0}$ and $V_{out}(t) = v_{out}(t) + V_{OUT,0}$.

The cut-off mode of operation corresponds to a collector current being equal to zero: $I_c(t) = 0$. Using the following equations:

$V_{CC} = R_C I_c(t) + R_E I_e(t) + V_{ce}(t) \approx (R_C + R_E) I_c(t) + V_{ce}(t)$ and $V_{CC} = R_C I_c(t) + V_{out}(t)$,

we can see that the BJT enters the cut-off mode once we have: (1) $I_c(t) = 0$, (2) $V_{ce}(t) = V_{CC}$, and (3) $V_{out}(t) = V_{CC}$.

The saturation mode of operation corresponds to a voltage $V_{ce}(t)$ being equal to $V_{CE,sat} \approx 0.2$ volt. Using the following equations:

$V_{CC} = R_C I_c(t) + R_E I_e(t) + V_{ce}(t) \approx (R_C + R_E) I_c(t) + V_{ce}(t)$ and $V_{CC} = R_C I_c(t) + V_{out}(t)$,

we can see that the BJT enters the saturation mode once we have:

(1) $I_c(t) \approx \frac{V_{CC} - V_{CE,sat}}{R_C + R_E} \approx \frac{V_{CC}}{R_C + R_E}$, (2) $V_{ce}(t) = V_{CE,sat}$, and (3) $V_{out}(t) \approx V_{CC} - \frac{R_C V_{CC}}{R_C + R_E} = \frac{R_E V_{CC}}{R_C + R_E}$.

To maximise the output voltage swing, we must make sure that the DC operating point is located midway between the start of the cut-off and saturation regions, i.e. we must have:

(1) $I_{C0} \approx \frac{1}{2}\left[0 + \frac{V_{CC}}{R_C + R_E}\right] = \frac{V_{CC}}{2\left(R_C + R_E\right)} \approx 0.45$ mA,

(2) $V_{CE0} \approx \frac{1}{2}\left[V_{CC} + V_{CE,sat}\right] \approx \frac{V_{CC}}{2} \approx 5$ V,

(3) $V_{OUT,0} \approx \frac{1}{2}\left[V_{CC} + \frac{R_E V_{CC}}{R_C + R_E}\right] = \frac{V_{CC}}{2}\left[\frac{R_C + 2R_E}{R_C + R_E}\right] \approx 5.45$ V.

The value of the DC collector current I_{C0} thus obtained leads to the desired value for the base current: $I_{B0} = \frac{I_{C0}}{\beta_F} \approx 4.5\ \mu$A.

Note that we can now compute the resistance h_{11} at room temperature as follows:

$$h_{11} \approx \frac{V_T}{I_{B0}} \approx \frac{25\text{ mV}}{4.5\ \mu\text{A}} \approx 5.6\text{ k}\Omega.$$

This confirms that the assumption $h_{11} << R_E \beta_F$ previously made can be considered as valid (as 5.6 kΩ << 100 kΩ).

We remember that the expression of the DC base current is given by $I_{B0} \approx \frac{V_{th} - 0.7}{R_{th} + R_E \beta_F} \approx 4.5\ \mu$A. Hence, we must choose the values of the quantities $V_{th} = \frac{R_2 V_{CC}}{R_1 + R_2}$ and $R_{th} = \frac{R_1 R_2}{R_1 + R_2}$ so that $I_{B0} \approx$ 4.5 μA. There are obviously several possible choices.

We also remember that the input resistance of the common-emitter amplifier with negative feedback is given by

$$r_{in} = \frac{\left[h_{11} + R_E\left(1 + \beta_F\right)\right]R_{th}}{h_{11} + R_E\left(1 + \beta_F\right) + R_{th}} \approx \frac{R_E \beta_F R_{th}}{R_E \beta_F + R_{th}}.$$

This expression clearly shows that, in order to obtain an input resistance r_{in} that is sufficiently large, we must make sure to select both resistances R_1 and R_2 so that R_{th} is not too small. On the other hand, R_{th} should not be too large either because it is crucial that the DC operating point does not significantly depend on the parameter β_F; indeed remember that the collector current is given by

$$I_{C0} \approx \beta_F \frac{V_{th} - 0.7}{R_{th} + R_E\,\beta_F}.$$

If $R_{th} << R_E\,\beta_F$, we have $I_{C0} \approx \frac{V_{th} - 0.7}{R_E}$, which is highly desirable in practice since the collector current is then completely independent from the parameter β_F.

We clearly have conflicting constraints once again!

In general, the stability of the DC operating point is the most crucial issue, i.e. we ensure that the condition $R_{th} << R_E\,\beta_F$ is satisfied.

In this case, we can write: $V_{th} \approx 0.7 + R_E I_{C0} \approx 0.7\text{ V} + 1\text{k}\Omega \times 0.45\text{ mA}$ $\approx$ 1.15 volt. In other words, we must select both resistances R_1 and R_2 so that $V_{th} = \frac{R_2\,V_{CC}}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} \times 10\text{ V}$ $\approx$ 1.15 volt, under the constraint $R_{th} = \frac{R_1 R_2}{R_1 + R_2} << R_E\beta_F = 100\text{ k}\Omega$.

We thus have: $\frac{R_2}{R_1 + R_2} \approx 0.115$, which yields $R_2 \approx 0.13 \times R_1$, with R_2 being equal, typically, to a few kΩs. For instance, we can choose R_2 = 3 kΩ and R_1 = 23 kΩ.

<u>Final checks (does it work?)</u>:

We have chosen the following values for the four resistances: R_1 = 23 kΩ, R_2 = 3 kΩ, R_C = 10 kΩ, and R_E = 1 kΩ.

With these values, we obtain:

- $V_{th} = \frac{R_2\,V_{CC}}{R_1 + R_2}$ $\approx$ 1.15 V;

- $R_{th} = \frac{R_1 R_2}{R_1 + R_2} \approx$ 2.65 kΩ. This result confirms that $R_{th} << R_E\beta_F = 100\,k\Omega$, thus guaranteeing an excellent stability of the DC operating point;
- $I_{C0} \approx \beta_F \frac{V_{th} - 0.7}{R_{th} + R_E\,\beta_F} \approx$ 0.44 mA (very close to the desired 0.45 mA) and, thus, $I_{B0} = \frac{I_{C0}}{\beta_F} \approx$ 4.4 μA (very close to the desired 4.5 μA);
- $V_{CE0} \approx V_{CC} - (R_C + R_E)I_{C0} \approx$ 5.2 V (very close to the desired 5 V) and $V_{OUT,0} \approx V_{CC} - R_C\,I_{C0} \approx$ 5.6 V (very close to the 5.45 V);
- $h_{11} \approx \frac{V_T}{I_{B0}} \approx \frac{25\,mV}{4.4\,\mu A} \approx$ 5.7 kΩ;
- $A_v \approx -\frac{\beta_F\,R_C}{h_{11} + R_E\,\beta_F} \approx$ -9.5 (very close to the desired voltage gain of 9);
- $r_{in} \approx \frac{[h_{11} + R_E\beta_F]R_{th}}{h_{11} + R_E\beta_F + R_{th}} \approx$ 2.6 kΩ, which means that the input resistance is "not too low";
- r_{out} = R_C = 10 kΩ, which means that the output resistance is "not too large".