BIT-INTERLEAVED TURBO-CODED MODULATIONS FOR MOBILE COMMUNICATIONS

Stéphane Yves Le Goff
Etisalat College of Engineering, PO Box 980, Sharjah, United Arab Emirates
E-mails: s_legoff@ece.ac.ae, s_legoff@hotmail.com

ABSTRACT

This paper deals with the study of bandwidth-efficient coding schemes for mobile communications. They are designed from the bit-interleaved coded modulation approach. Since turbo coding technique is utilized for correcting errors, these schemes are termed bit-interleaved turbo-coded modulations (BITCMs). Their structure is presented in a detailed manner and their bit error rate (BER) performance over Rayleigh fading channels is investigated for spectral efficiencies ranging from 2 to 7 bit/s/Hz. Simulation results show that BITCMs are attractive for mobile communications and, in particular, achieve performance being within 1.1 dB of their respective channel capacity at a BER of 10^{-5}.

1. INTRODUCTION

Turbo codes are revolutionary error-correcting codes introduced in 1993 by Berrou *et al.* [1]. In essence, they are parallel concatenated convolutional codes. The turbo encoder is usually constructed from two recursive systematic convolutional (RSC) encoders connected in parallel and separated by a pseudo-random interleaver. A maximum likelihood (ML) decoding of a turbo code is impractical because of the computational complexity. Therefore, the decoder uses an iterative, sub-optimal, soft-decoding rule where each constituent RSC is decoded separately. The constituent decoders participate in sharing of bit-likelihood information in an iterative process. Usually, these decoders employ the soft-input/soft-output MAP algorithm. Although the global turbo decoder is not ML, it achieves bit error rate (BER) performance that is close to the Shannon limit after a few iterations. For instance, in [1], operation within 0.7 dB of the Shannon limit is achieved on an additive white Gaussian noise (AWGN) channel.

In order to design bandwidth-efficient coding schemes, some successful attempts were recently undertaken to combine turbo codes with multilevel modulations over AWGN channels [2-4]. For mobile communications requiring large coding gain and high bandwidth efficiency over fading channels, it is also interesting to study BER performance of high coding rate turbo codes combined with high order QAM modulations.

In this paper, we investigate the performance of several bandwidth-efficient schemes designed from the bit-interleaved coded modulation (BICM) approach, which has been proven to be particularly attractive over Rayleigh fading channels [5]. BICM is basically a serial concatenation of a binary error-correcting code with a high order modulation. The major characteristic of BICM is that coding and modulation functions are optimized separately. In 1992, Zehavi showed that, on Rayleigh fading channels, this separation of modulation and coding can be beneficial, provided that the encoder output is bit-by-bit interleaved and relevant soft-decisions are used by the binary decoder [6]. As turbo coding technique is here utilized for correcting errors, the systems studied in this paper are termed 'bit-interleaved turbo-coded modulation' (BITCM).

The paper begins by describing the generic structure of the BITCM transmitter and receiver. Then, we present some simulation results of several BITCMs employing turbo codes whose coding rates R vary from 1/2 up to 7/8, combined with square M-QAM constellations (M = 16, 64, and 256). Spectral efficiencies η are thus in the range from 2 to 7 bit/s/Hz.

Results are obtained by considering a Rayleigh fading channel with perfect channel state information (CSI). Rician channel could be a better model for mobile communications if there is LOS (line-of-sight), but when an omni-directional antenna is used and LOS is blocked by trees, poles, or buildings, Rayleigh fading can be used as a worst-case scenario.

2. BITCM TRANSMITTER STRUCTURE

The block diagram of a BITCM transmitter is depicted in Fig.1. This transmitter is made up of an M-state ($M = 2^m$) QAM modulator (MOD) combined with a rate R encoder built from a standard rate-1/3 turbo encoder (ENC) by using puncturing technique. Only redundant bits are punctured since deletion of systematic bits leads to inferior BER performance for the iterative decoder. In order to obtain symbols affected with uncorrelated noises at the turbo decoder input, an interleaver π is inserted between the puncturer and the modulator.

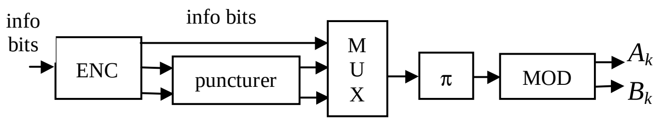

Fig. 1 – Structure of the generic BITCM transmitter.

With this approach, the spectral efficiency η of the transmission is given by $\eta = R \,\mathrm{Log}_2 M$. At time k, each point of the QAM constellation is represented by a couple of realvalued symbols $\{A_k, B_k\}$ coded by a set $\{u_{k,i}\}$, $i \in \{1 \ldots m\}$, of m bits according to Gray mapping. With such a mapping, the bits $u_{k,i}$ are not equally protected. For instance, Fig. 2 shows the Gray mapping corresponding to the symbol A_k (I component) for a square 16-QAM constellation. In this example, the 'most protected bit' $u_{k,2}$ has higher error margin than the 'least protected bit' $u_{k,1}$, which is translated in different error probabilities for each bit.

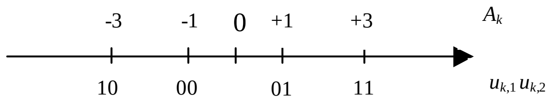

Fig. 2 – Gray mapping for the I component of a square 16-QAM constellation.

In a set $\{u_{k,i}\}$, some bits represent information bits, while the other bits correspond to redundant bits. Our simulations have indicated that the best BER performance is obtained when the information bits are associated with the most protected bits in the set $\{u_{k,i}\}$. It can be briefly justified as follows: we have noticed that the iterative decoding process starts being efficient mainly when the BER on the information sequence at the decoder input reaches a certain level. In other words, the signal-to-noise ratio (SNR) at which the turbo decoding starts performing well depends essentially on the BER of the information sequence, and is less influenced by the BER of the redundant sequence. Thus, in order to ensure that this SNR is as low as possible, it is necessary to offer a maximum protection to information bits.

3. BITCM RECEIVER STRUCTURE

3.1 Generic receiver structure

In this paper, we consider a transmission over a nondispersive Rayleigh slow-fading channel, which is a good model for mobile communications. Assuming coherent detection, the decoder receives, at time k, two samples X_k and Y_k modeled with the following expressions:

$$X_k = \alpha_{k,1} A_k + n_{k,1}$$
$$Y_k = \alpha_{k,2} B_k + n_{k,2} \qquad (1)$$

where $\alpha_{k,1}$ and $\alpha_{k,2}$ are two Rayleigh random variables. The samples $n_{k,1}$ and $n_{k,2}$ are two uncorrelated AWGN components with zero mean, variance σ^2, and independent of samples $\alpha_{k,i}$. A fading amplitude $\alpha_{k,i}$ is modeled with a Rayleigh pdf. With sufficient channel interleaving, samples $\alpha_{k,i}$ are independent. In addition, a perfect CSI is assumed.

The structure of a generic BITCM receiver is depicted in Fig. 3. From samples X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$, the logarithm of likelihood ratio (LLR) $\Lambda(u_{k,i})$ associated with each bit $u_{k,i}$, $i \in \{1 \ldots m\}$, is computed and used as a relevant soft-decision by the turbo decoder. This operation is fundamental since this turbo decoder is a binary decoder optimized for AWGN channels, and therefore not able to process samples X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$ coming from the channel.

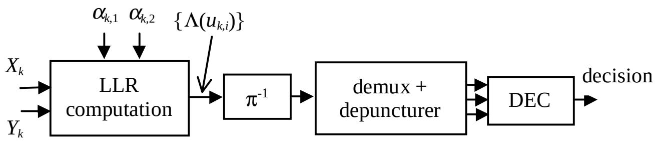

Fig. 3 – BITCM receiver structure.

The LLRs $\Lambda(u_{k,i})$ are obtained by using the relation, for $i \in \{1 \ldots m\}$:

$$\Lambda(u_{k,i}) = K \, Log_e \frac{P_r\left(u_{k,i} = 1 / X_k, Y_k, \alpha_{k,1}, \alpha_{k,2}\right)}{P_r\left(u_{k,i} = 0 / X_k, Y_k, \alpha_{k,1}, \alpha_{k,2}\right)} \qquad (2)$$

where K is a constant, and P_r $(u_{k,i} = j \,/\, X_k, Y_k, \alpha_{k,1}, \alpha_{k,2})$ is the probability that $u_{k,i} = j$ ($j \in \{0, 1\}$) given samples X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$.

Applying Bayes'rule twice from the previous expression and assuming that P_r $(u_{k,i} = 1) = P_r$ $(u_{k,i} = 0)$, we can also write:

$$\Lambda(u_{k,i}) = K \, Log_e \frac{P_r\left(X_k, Y_k / u_{k,i} = 1, \alpha_{k,1}, \alpha_{k,2}\right)}{P_r\left(X_k, Y_k / u_{k,i} = 0, \alpha_{k,1}, \alpha_{k,2}\right)} \qquad (3)$$

where P_r $(X_k, Y_k \,/\, u_{k,i} = j, \alpha_{k,1}, \alpha_{k,2})$ is the probability of the couple $\{X_k, Y_k\}$ given the couple $\{\alpha_{k,1}, \alpha_{k,2}\}$ and assuming that bit $u_{k,i}$ is equal to j.

At time k, the m LLRs $\Lambda(u_{k,i})$ are correlated since they are obtained from the same samples X_k, Y_k, $\alpha_{k,1}$, and $\alpha_{k,2}$. It is therefore necessary to use a deinterleaver π^{-1}, associated with the interleaver used at the encoder side, which suppresses correlation between quantities $\Lambda(u_{k,i})$ and thus ensures an efficient turbo decoding.

The standard binary turbo decoder (DEC) must be fed with three samples at each time k, since its associated encoder has a coding rate of 1/3. The punctured bits are replaced with zero (neutral) values at the input of the turbo decoder. This operation is performed by the depuncturer.

3.2 Use of square *M*-QAM constellations

Consider now that the modulation scheme is an M-QAM with square constellation. Since Gray mapping is used, such a modulation consists of two identical $M^{1/2}$-ASK modulations in quadrature. Symbol A_k is associated with the subset $\{u_{k,i}\}$, $i \in \{1 \ldots m/2\}$ while symbol B_k is associated with the subset $\{u_{k,i}\}$, $i \in \{(m/2)+1 \ldots m\}$. Let us first consider the I component. The LLRs $\Lambda(u_{k,i})$, $i \in \{1 \ldots m/2\}$, only depend on observations X_k and $\alpha_{k,1}$, and can therefore be obtained using a simplified version of (3):

$$\Lambda(u_{k,i}) = K \, Log_e \frac{P_r\left(X_k / u_{k,i} = 1, \alpha_{k,1}\right)}{P_r\left(X_k / u_{k,i} = 0, \alpha_{k,1}\right)} \qquad (4)$$

which yields

$$\Lambda(u_{k,i}) = K\, Log_e \frac{\sum_{n=1}^{n=2^{(m-2)/2}} \exp\left\{ -\frac{(X_k - \alpha_{k,1} a_{n,1})^2}{2\sigma^2} \right\}}{\sum_{n=1}^{n=2^{(m-2)/2}} \exp\left\{ -\frac{(X_k - \alpha_{k,1} a_{n,0})^2}{2\sigma^2} \right\}} \quad (5)$$

where variables $a_{n,j}$ are the possible realizations of symbol A_k conditionally on $u_{k,i} = j$ ($j \in \{0, 1\}$). Unfortunately, the computation of $m/2$ quantities $\Lambda(u_{k,i})$ from (5) is quite complex, and also requires the knowledge of the noise variance σ^2.

3.2.1 Square 16-QAM constellation

It is therefore of interest to simplify equation (5). Consider for example the case of a 16-QAM modulation with Gray mapping. Taking $K = \sigma^2/2$ and performing some minor simplifications from (5), we can demonstrate that the LLR $\Lambda(u_{k,1})$ is well approximated by

$$\Lambda(u_{k,1}) \approx \alpha_{k,1} (\,|X_k| - 2\alpha_{k,1}) \qquad (6)$$

As for the LLR $\Lambda(u_{k,2})$, we can also show that an accurate and simple approximation of (5) is

$$\Lambda(u_{k,2}) \approx \alpha_{k,1} X_k \qquad (7)$$

Regarding the Q component, LLRs $\Lambda(u_{k,3})$ and $\Lambda(u_{k,4})$ depend only on observations Y_k and $\alpha_{k,2}$, and can be expressed with relations (6) and (7), respectively, but with Y_k substituted for X_k, and $\alpha_{k,2}$ for $\alpha_{k,1}$.

3.2.2 General case: square *M*-QAM constellation

Following the same reasoning as in the previous subsection, we can show that LLRs $\Lambda(u_{k,i})$ for any M-QAM modulation with square constellation can be well approximated with relations

$$\begin{aligned}
&\Lambda(u_{k,1}) \approx \alpha_{k,1} (\,|X_k| - \alpha_{k,1} \times 2^{(m-2)/2}) \\
&\ldots \\
&\Lambda(u_{k,i}) \approx |\Lambda(u_{k,i-1})| - (\alpha_{k,1})^2 \times 2^{(m-2i)/2} \\
&\ldots \qquad \text{for } i \in \{2\ldots(m-2)/2\} \qquad (8) \\
&\Lambda(u_{k,m/2}) \approx \alpha_{k,1} X_k
\end{aligned}$$

and

$$\begin{aligned}
&\Lambda(u_{k,(m+2/2)}) \approx \alpha_{k,2} (\,|Y_k| - \alpha_{k,2} \times 2^{(m-2)/2}) \\
&\ldots \\
&\Lambda(u_{k,(m+2i)/2)}) \approx |\Lambda(u_{k,(m+2i-2)/2)})| - (\alpha_{k,2})^2 \times 2^{(m-2i)/2} \\
&\ldots \qquad \text{for } i \in \{2\ldots(m-2)/2\} \qquad (9) \\
&\Lambda(u_{k,m}) \approx \alpha_{k,2} Y_k
\end{aligned}$$

The deinterleaver π^{-1} inserted at the LLR computation module output breaks the correlation introduced by the modulation. The fact that samples $\Lambda(u_{k,i})$ are just correlated over $m/2$ values allows us to employ a short interleaving function, typically made up of a matrix with $m/2$ rows and a number of columns equal to several times the constraint length of a constituent encoder. Based on simulation results, we have noticed that absence of interleaving significantly degrades the BER at the turbo decoder output. However, a short interleaver is sufficient to approach the ultimate performance.

4. SIMULATIONS

In this paper, we examine performance at moderate BERs based on Monte Carlo simulations. As examples, we use several BITCMs providing spectral efficiencies ranging from 2 up to 7 bit/s/Hz, and employing square 16-QAM, 64-QAM, and 256-QAM constellations. Simulations consider standard rate-1/3 turbo code made up of two 16-state RSC codes with generators (23, 31). The size of the pseudo-random interleaver is 65536 bits. Turbo decoding is performed in either 5 or 10 iterations from the LLRs $\Lambda(u_{k,i})$ given by (8) and (9). The MAP algorithm is used for the decoding of each constituent RSC code. The BER curves are shown in Fig. 4 for 16-QAM, in Fig. 5 for 64-QAM, and finally in Fig. 6 for 256-QAM. For each BITCM scheme, the capacity limit has been computed using some results given in [6].

From Figs. 4, 5, and 6, it is seen that BITCMs perform very well over Rayleigh channels for all spectral efficiencies. In particular, BER performance is very close to channel capacity in all cases. For instance, at a BER of 10^{-5}, the gap between the capacity and the performance of a BITCM built from 16-QAM and rate-1/2 turbo code is equal to 1.4 dB with 5 iterations, 1.1 dB with 10 iterations, and only 0.95 dB when a very large number of iterations is used. For any BITCM, we have noticed that BER performance is within 1.1 dB of the channel capacity, provided that the number of iterations is large enough.

It is also possible to use these BITCM schemes over AWGN channels. For this purpose, samples $\alpha_{k,i}$ must just be replaced with values '1' at the input of the LLR computation module. In this case, BER performance still remains excellent.

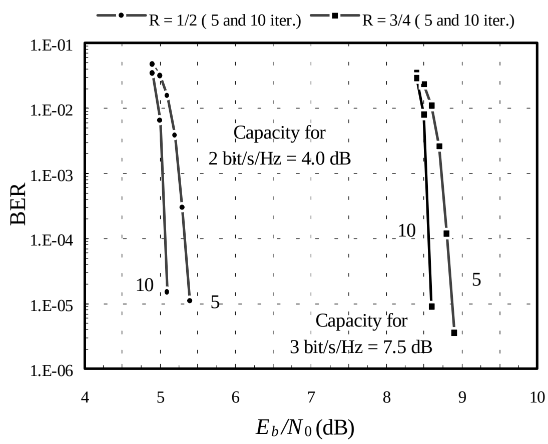

Fig. 4 – BER performance over Rayleigh channel of two BITCMs using 16-QAM and a turbo code with coding rate R = 1/2 or 3/4.

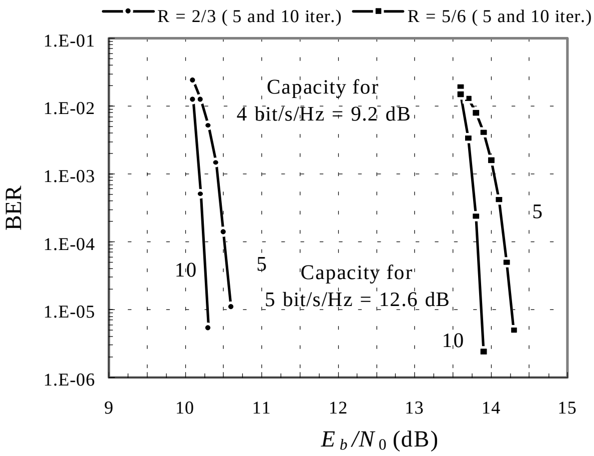

Fig. 5 – BER performance over Rayleigh channel of two BITCMs using 64-QAM and a turbo code with coding rate R = 2/3 or 5/6.

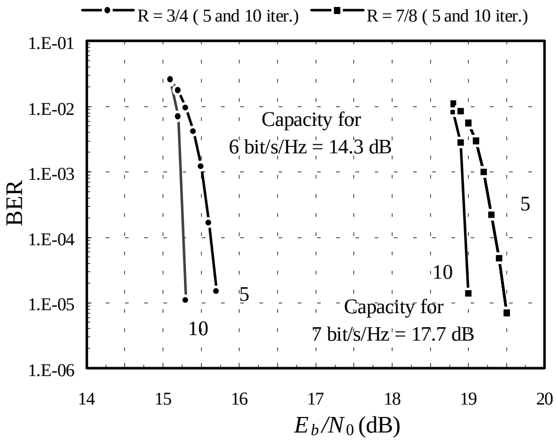

Fig. 6 – BER performance over Rayleigh channel of two BITCMs using 256-QAM and a turbo code with coding rate R = 3/4 or 7/8.

In [7], it was shown that the BICM approach achieves the same performance on AWGN channels as that obtained using Ungerboeck's coded modulation (CM) approach [8] applied to turbo codes [3, 4], although the latter is suggested to be superior over these channels. Since BICMs are potentially more attractive than CMs on Rayleigh fading channels [6], it is our belief that the application of CM principles to turbo codes will not lead to better performance than that achieved with BITCMs, when a mobile communications application is considered. However, further study concerning this point is necessary.

5. CONCLUSIONS

BER performance of several BITCMs has been analyzed for mobile communications applications. Although they are easy to implement and quite flexible (the same standard turbo code is employed for different spectral efficiencies), BITCMs achieve a performance which is very close to the capacity on Rayleigh fading channels. In addition, BITCMs are very attractive for high-speed communications since only one binary turbo decoder utilizing RSC constituent decoders with a small number of states (in practice, 8 or 16) is needed.

6. REFERENCES

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