ENG1004 - Electronics & Sensors

Tutorial 2 – BJT Circuits – Solutions

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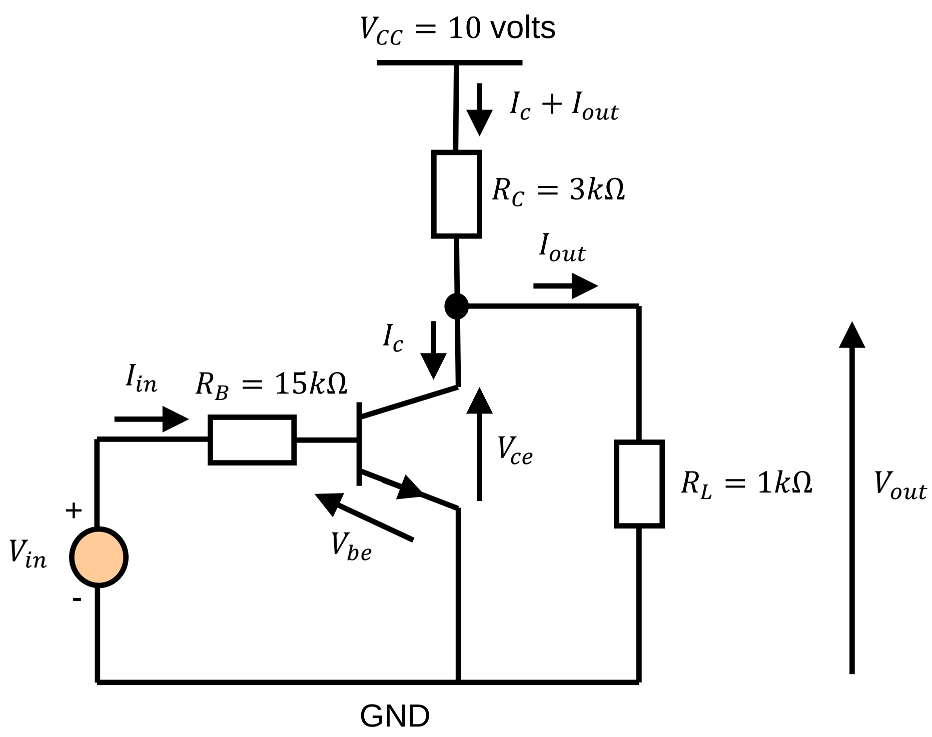
Question 1



(a) Let us write the general equations for the common-emitter circuit, using Kirchhoff voltage law (KVL) and Ohm's law:

(1) Using KVL, $V_{out} = V_{ce}$;

(2) Using KVL and Ohm's law, $V_{be} + R_B \cdot I_{in} = V_{in}$;

(3) Using KVL and Ohm's law, $V_{out} + R_C \cdot (I_c + I_{out}) = V_{CC}$;

(4) Using Ohm's law, $V_{out} = R_L \cdot I_{out}$.

We thus obtain a set of four equations that are always valid, regardless the mode in which the BJT operates. Now, we can move to the next step: consider the three possible modes of operation and examine what happens to the general equations.

(b) First mode of operation: Cut-off mode if $V_{be} < V_{BE,on}$ and $I_{in} = I_c = 0$.

In this mode of operation, Equation 1, $V_{out} = V_{ce}$, remains unchanged.

Equation 2, $V_{be} + R_B \cdot I_{in} = V_{in}$, becomes $V_{be} = V_{in}$ because $I_{in} = 0$. This result yields $V_{in} < V_{BE,on}$ as $V_{be} < V_{BE,on}$.

The inequality $V_{in} < V_{BE,on}$ provides us with a condition on the input voltage V_{in} for the BJT to be cut-off.

Equation 3, $V_{out} + R_C \cdot (I_c + I_{out}) = V_{CC}$, leads to $V_{out} + R_C \cdot I_{out} = V_{CC}$ because $I_c = 0$.

Equation 4, $V_{out} = R_L \cdot I_{out}$, remains unchanged.

To obtain the expression of V_{out}, we can combine Equations 3 and 4 as follows: $V_{out} + R_C \cdot I_{out} = V_{CC}$ can be written as $V_{out} + R_C \cdot \frac{V_{out}}{R_L} = V_{CC}$. This leads to $V_{out}\left(1 + \frac{R_C}{R_L}\right) = V_{CC}$, and finally to $V_{out} = \frac{R_L V_{CC}}{R_L + R_C} = 2.5$ volts.

Let us now determine the expressions and values of the input and output currents for the cut-off mode. We can see that $I_{in} = 0$ and $I_{out} = \frac{V_{out}}{R_L} = \frac{V_{CC}}{R_L + R_C} = 2.5$ mA.

Conclusion: when $V_{in} < V_{BE,on}$, the BJT is cut-off and we have $V_{out} = \frac{R_L V_{CC}}{R_L + R_C} = 2.5$ volts. As for the currents, we can also write $I_{in} = 0$ and $I_{out} = \frac{V_{CC}}{R_L + R_C} = 2.5$ mA.

(c) Second mode of operation: Forward-active mode if $V_{be} = V_{BE,on}$ and $V_{ce} > V_{CE,sat}$. In this case, we have $I_c = \beta_F \cdot I_{in}$, $I_{in} > 0$, and $I_c > 0$.

In the forward-active mode, Equation 1, $V_{out} = V_{ce}$, yields $V_{out} > V_{CE,sat}$ because $V_{ce} > V_{CE,sat}$.

Equation 2, $V_{be} + R_B \cdot I_{in} = V_{in}$, becomes $V_{BE,on} + R_B \cdot I_{in} = V_{in}$ as $V_{be} = V_{BE,on}$, which leads to $V_{BE,on} < V_{in}$ as $I_{in} > 0$.

Equation 3, $V_{out} + R_C \cdot (I_c + I_{out}) = V_{CC}$, leads to $V_{out} + R_C \cdot (\beta_F \cdot I_{in} + I_{out}) = V_{CC}$, i.e. $V_{out} + R_C \cdot \beta_F \cdot I_{in} + R_C \cdot I_{out} = V_{CC}$, since $I_c = \beta_F \cdot I_{in}$.

So, at this stage, we have discovered that the BJT is forward active when $V_{in} > V_{BE,on}$ and $V_{out} > V_{CE,sat}$.

But we need to know more than that. Remember that our goal is to express V_{out} as a function of V_{in}, and also I_{out} as a function of I_{in}. This can be done by further exploiting Equations (2), (3), and (4) as follows.

Equation (3) can be written as $V_{out} + R_C \cdot I_{out} = V_{CC} - R_C \cdot \beta_F \cdot I_{in}$. The unknowns in this equation are the currents I_{in} and I_{out} because the values of the circuit parameters V_{CC}, R_C, and β_F are known.

Using Equation (2), we can obtain an expression of I_{in} as a function of V_{in}:

$$I_{in} = \frac{V_{in} - V_{BE,on}}{R_B}.$$

Using Equation (4), we can obtain an expression of I_{out} as a function of V_{out}:

$$I_{out} = \frac{V_{out}}{R_L}.$$

Equation (3) finally leads to

$$V_{out}\left(1 + \frac{R_C}{R_L}\right) = V_{CC} - R_C \cdot \beta_F \cdot \frac{V_{in} - V_{BE,on}}{R_B} = V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{in} - V_{BE,on}\right).$$

We finally obtain $V_{out} = \frac{R_L}{R_L + R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{in} - V_{BE,on}\right)\right]$.

This shows that V_{out} varies linearly with V_{in}. The slope of this linear function is negative and given by $-\beta_F \cdot \frac{R_C}{R_B} \cdot \frac{R_L}{R_L + R_C} = -5$.

As for the output current expression, we can write

$$I_{out} = \frac{V_{out}}{R_L} = \frac{1}{R_L+R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{in} - V_{BE,on}\right)\right],$$

with, according to Equation (2), $I_{in} = \frac{V_{in}-V_{BE,on}}{R_B}$.

Hence, we finally obtain $I_{out} = \frac{1}{R_L+R_C} \cdot [V_{CC} - \beta_F \cdot R_C \cdot I_{in}]$.

This shows that the output current I_{out} varies linearly with I_{in}. The slope of this linear function is negative and given by $-\beta_F \cdot \frac{R_C}{R_L+R_C} = -75$.

We have obtained two conditions that need to be satisfied for the BJT to be forward active: $V_{in} > V_{BE,on}$ and $V_{out} > V_{CE,sat}$. The second condition is not useful as such because what we want are conditions on the input voltage V_{in} only.

Since we now know that $V_{out} = \frac{R_L}{R_L+R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{in} - V_{BE,on}\right)\right]$, we can replace the condition $V_{out} > V_{CE,sat}$ with another condition $V_{in} < V_{in,sat}$, where the voltage $V_{in,sat}$ is obtained using

$$V_{CE,sat} = \frac{R_L}{R_L+R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{in,sat} - V_{BE,on}\right)\right].$$

This expression leads to $V_{in,sat} = V_{BE,on} + \frac{R_B}{\beta_F R_C} \cdot \left(V_{CC} - \frac{R_L+R_C}{R_L} V_{CE,sat}\right) = 1.16$ volts.

We are also asked to find a condition on I_{in} rather than V_{in}. To do so, we can simply remember the expression $I_{in} = \frac{V_{in}-V_{BE,on}}{R_B}$. The condition $V_{BE,on} < V_{in} < V_{in,sat}$ is actually equivalent to

$$\frac{V_{BE,on}-V_{BE,on}}{R_B} < \frac{V_{in}-V_{BE,on}}{R_B} < \frac{V_{in,sat}-V_{BE,on}}{R_B},$$

which yields

$$0 < I_{in} < I_{in,sat},$$

with $I_{in,sat} = \frac{1}{\beta_F R_C} \cdot \left(V_{CC} - \frac{R_L+R_C}{R_L} V_{CE,sat}\right) \sim 30.7$ microamperes (μA).

Conclusion: when $V_{BE,on} < V_{in} < V_{in,sat}$, the BJT is forward active and we have $V_{out} = \frac{R_L}{R_L+R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot (V_{in} - V_{BE,on})\right]$. Alternatively, if we consider currents rather than voltages, we can write that, when $0 < I_{in} < I_{in,sat}$, the BJT is forward active and we have $I_{out} = \frac{1}{R_L+R_C} \cdot [V_{CC} - \beta_F \cdot R_C \cdot I_{in}]$.

(d) Third mode of operation: Saturation mode if $V_{be} = V_{BE,on}$ and $V_{ce} = V_{CE,sat}$. In this case, we have $I_{in} > 0$, and $I_c > 0$.

The last possible mode of operation, the saturation mode, is the easiest to process. From the results that have been previously obtained, it is indeed very easy to determine the condition(s) on V_{in} that need(s) to be satisfied for the BJT to be saturated.

Since the three modes of operation are mutually exclusive, the BJT must be saturated as long as it is neither cut-off nor forward-active. We conclude that the BJT is saturated when $V_{in} > V_{in,sat}$ or, equivalently, $I_{in} > I_{in,sat}$.

In the saturation mode, Equation 1, $V_{out} = V_{ce}$, becomes $V_{out} = V_{CE,sat}$ as $V_{ce} = V_{CE,sat}$.

Equation 1 alone is sufficient to provide us with the expression of the output voltage V_{out}: $V_{out} = V_{CE,sat} = 0.2$ volt.

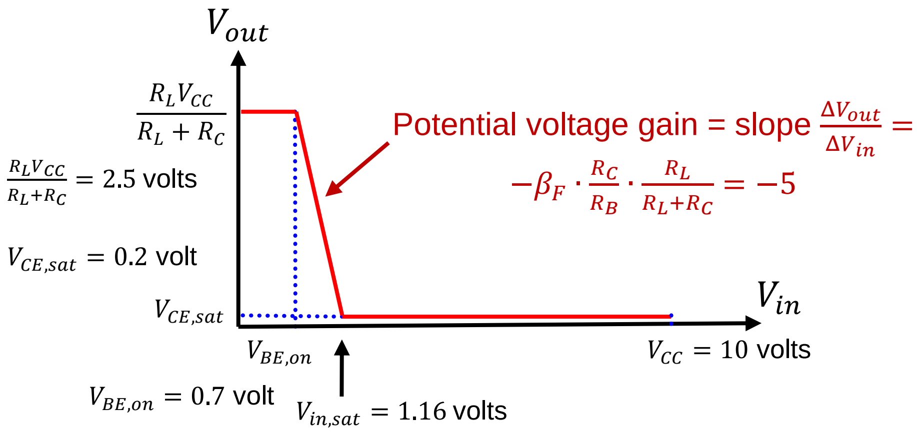
Equation 4, $V_{out} = R_L \cdot I_{out}$, can then be used to express the output current as

$$I_{out} = \frac{V_{out}}{R_L} = \frac{V_{CE,sat}}{R_L} = 0.2 \text{ mA}.$$

Conclusion: when $V_{in} > V_{in,sat}$, the BJT is saturated and we have $V_{out} = V_{CE,sat} = 0.2$ volt. Alternatively, if we choose to think in terms of currents instead, we can write that, when $I_{in} > I_{in,sat}$, the BJT is saturated and we have $I_{out} = \frac{V_{CE,sat}}{R_L} = 0.2$ mA.

We can now plot the two DC transfer characteristics corresponding to the previous results.

(e) The DC voltage transfer characteristic is shown below.



The voltage $V_{in,sat}$ corresponds to the value of the input voltage at which the BJT leaves the forward-active mode to enter the saturation mode.

We can observe the potential of the common-emitter circuit as a linear voltage amplifier when the BJT is forward active because, in that mode, the output voltage V_{out} is a linear function of the input voltage V_{in}: $V_{out} = \frac{R_L}{R_L+R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot (V_{in} - V_{BE,on})\right]$.

In the lecture notes on BJTs, it was shown that the voltage gain A_v of the amplifier would be the slope $\frac{\Delta V_{out}}{\Delta V_{in}}$ of the DC voltage transfer characteristic in the forward-active region, i.e. we would have

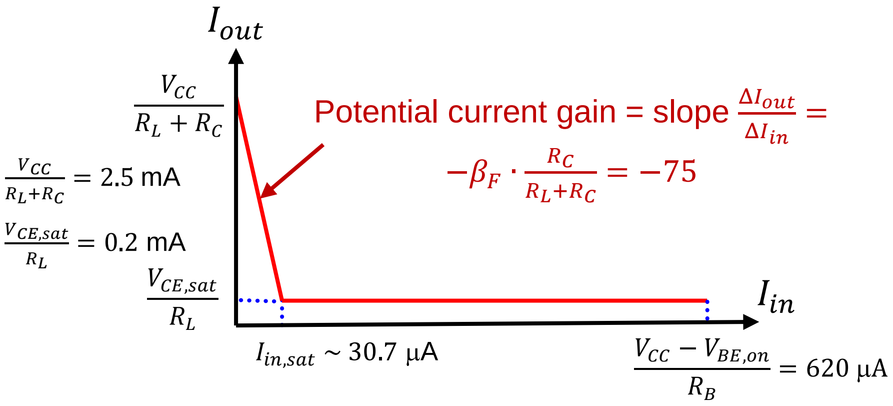
$$A_v = -\beta_F \cdot \frac{R_C}{R_B} \cdot \frac{R_L}{R_L+R_C} = -5.$$

The presence of a minus sign in the voltage gain means that the variations in V_{out} and V_{in} are in opposite directions, which is generally not an issue in practice.

(f) Let us now plot the DC current transfer characteristic obtained for the common-emitter circuit. The variation of I_{out} as a function of I_{in} is shown below.

The current $I_{in,sat}$ corresponds to the value of the input current at which the BJT leaves the forward-active mode to enter the saturation mode.

The maximum possible value of the input current I_{in} is obtained when $V_{in} = V_{CC}$, and therefore given by $I_{in} = \frac{V_{CC}-V_{BE,on}}{R_B} = 620\ \mu\text{A}$.



We can observe the potential of the common-emitter circuit as a current amplifier when the BJT is forward active because, in that mode, the output current I_{out} is a linear function of the input current I_{in}: $I_{out} = \frac{1}{R_L+R_C} \cdot [V_{CC} - \beta_F \cdot R_C \cdot I_{in}]$.

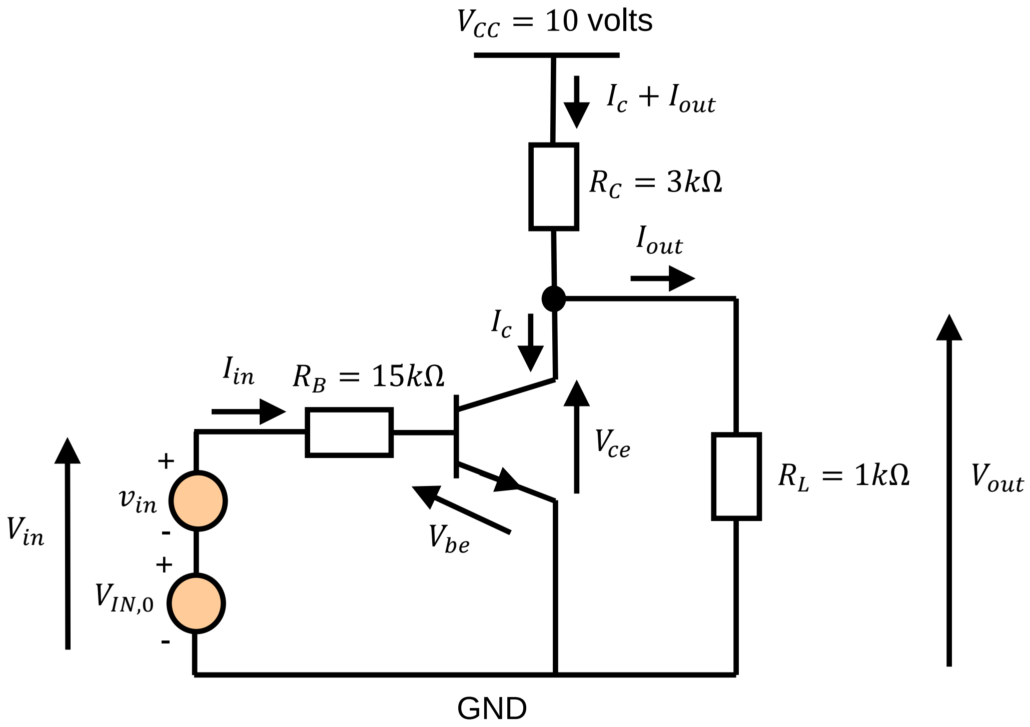
The current gain A_i of this amplifier would be the slope $\frac{\Delta I_{out}}{\Delta I_{in}}$ of the DC current transfer characteristic in the forward-active region, i.e., we can write $A_i = -\beta_F \cdot \frac{R_C}{R_L+R_C} = -75$.

The presence of a minus sign in the current gain expression means that the variations in I_{out} and I_{in} are in opposite directions, which is generally not an issue in practice.

To make our circuit work as a linear amplifier in practice, we will have to make sure that the BJT remains at all times in the forward-active mode and enters neither saturation nor cut-off mode. This issue is going to be addressed in Question 2.

Question 2

We can modify the circuit studied in Question 1 in order to design a linear common-emitter amplifier that can amplify both a voltage and a current. The modified circuit is shown in the figure below.



The input voltage V_{in} is now composed of two voltage sources $V_{IN,0}$ and v_{in} connected in series. Hence, we can write $V_{in} = V_{IN,0} + v_{in}$.

The DC voltage source $V_{IN,0}$ is used to bias the circuit so that the BJT operates at all times in the forward-active mode. The quantity $V_{IN,0}$ is referred to as bias voltage and its value has to be chosen very carefully by the designer. Remember that a BJT has the ability to amplify a current or voltage only when it operates in the forward-active mode.

The AC voltage source v_{in} generates the AC voltage to be amplified. It is perfectly reasonable to assume that v_{in} is a pure AC voltage signal with zero mean and small-amplitude symmetrical swings around zero volt.

Throughout this question, we are going to assume that the BJT operates in the forward-active mode. In other words, we are going to assume that the condition $V_{BE,on} < V_{in} < V_{in,sat}$ remains satisfied at all times.

(a) Since we have assumed that the BJT operates at all times in the forward-active mode, we can write

$$V_{in} = V_{BE,on} + R_B \cdot I_{in},$$

which can be written as $I_{in} = \frac{V_{in} - V_{BE,on}}{R_B} = \frac{V_{IN,0} + v_{in} - V_{BE,on}}{R_B} = \frac{V_{IN,0} - V_{BE,on}}{R_B} + \frac{v_{in}}{R_B} = I_{IN,0} + i_{in}$.

This expression indicates that the input current, I_{in}, can be written as the sum of a DC current, given by $I_{IN,0} = \frac{V_{IN,0} - V_{BE,on}}{R_B}$, and an AC current, given by $i_{in} = \frac{v_{in}}{R_B}$. So, we have $I_{in} = I_{IN,0} + i_{in}$.

(b) The results obtained in Question 1 can be applied to our circuit. It was shown that, as long as the BJT operates in the forward active mode, i.e., if $V_{BE,on} < V_{in} < V_{in,sat}$, the output voltage V_{out} is given by

$$V_{out} = \frac{R_L}{R_L + R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{in} - V_{BE,on}\right)\right],$$

which yields

$$V_{out} = \frac{R_L}{R_L + R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{IN,0} - V_{BE,on}\right)\right] - \beta_F \cdot \frac{R_C}{R_B} \cdot \frac{R_L}{R_L + R_C} \cdot v_{in}.$$

The expression of V_{out} means that we can express the output voltage as follows:

$$V_{out} = V_{OUT,0} + v_{out},$$

where

- $V_{OUT,0} = \frac{R_L}{R_L + R_C} \cdot \left[V_{CC} - \beta_F \cdot \frac{R_C}{R_B} \cdot \left(V_{IN,0} - V_{BE,on}\right)\right]$ represents the DC component of V_{out};
- $v_{out} = -\beta_F \cdot \frac{R_C}{R_B} \cdot \frac{R_L}{R_L + R_C} \cdot v_{in}$ is an amplified replica of the input AC voltage.

The output voltage V_{out} is thus composed of a DC component, $V_{OUT,0}$, and an AC component, v_{out}. The AC component v_{out} is an amplified version of the AC component v_{in} of the input voltage.

(c) The voltage gain of the circuit is defined as

$$A_v = \frac{v_{out}}{v_{in}},$$

and is thus given here by

$$A_v = -\beta_F \cdot \frac{R_C}{R_B} \cdot \frac{R_L}{R_L + R_C} = -5,$$

since $\beta_F = 100$, $R_B = 15\ k\Omega$, $R_L = 1\ k\Omega$, and $R_C = 3\ k\Omega$.

(d) The results obtained in Question 1 can be applied to our circuit. It was shown that, as long as the BJT operates in the forward active mode, i.e., if $0 < I_{in} < I_{in,sat}$, the output current I_{out} can be expressed as

$$I_{out} = \frac{1}{R_L + R_C} \cdot (V_{CC} - \beta_F \cdot R_C \cdot I_{in}),$$

which yields $I_{out} = \frac{V_{CC} - \beta_F R_C I_{IN,0}}{R_L + R_C} - \beta_F \cdot \frac{R_C}{R_L + R_C} \cdot i_{in}$.

The expression of I_{out} means that we can express the output current as follows:

$$I_{out} = I_{OUT,0} + i_{out},$$

where

- $I_{OUT,0} = \frac{V_{CC} - \beta_F R_C I_{IN,0}}{R_L + R_C}$ represents the DC component of I_{out};
- $i_{out} = -\beta_F \cdot \frac{R_C}{R_L + R_C} \cdot i_{in}$ is an amplified replica of the input AC current.

The output current I_{out} is thus composed of a DC component, $I_{OUT,0}$, and an AC component, i_{out}, the latter being an amplified version of the AC component i_{in} of the input current.

(e) The current gain of the common-emitter circuit is defined as

$$A_i = \frac{i_{out}}{i_{in}},$$

and is given here by

$$A_i = -\beta_F \cdot \frac{R_C}{R_L + R_C} = -75,$$

since $\beta_F = 100$, $R_L = 1\ k\Omega$, and $R_C = 3\ k\Omega$.

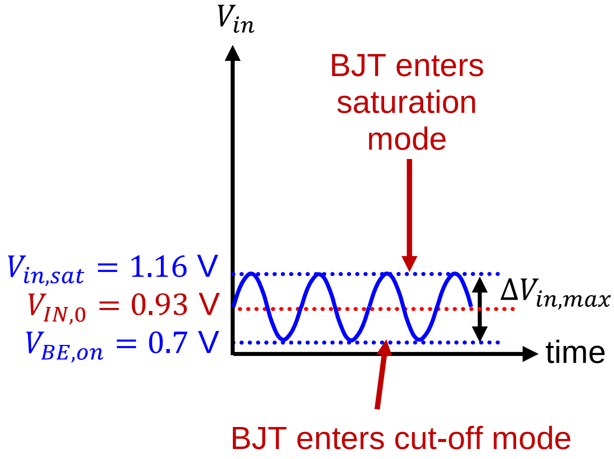
The results obtained in (c) and (e) clearly show that our common-emitter circuit behaves simultaneously as a voltage amplifier with a gain $A_v = -5$ and a current amplifier with a gain $A_i = -75$.

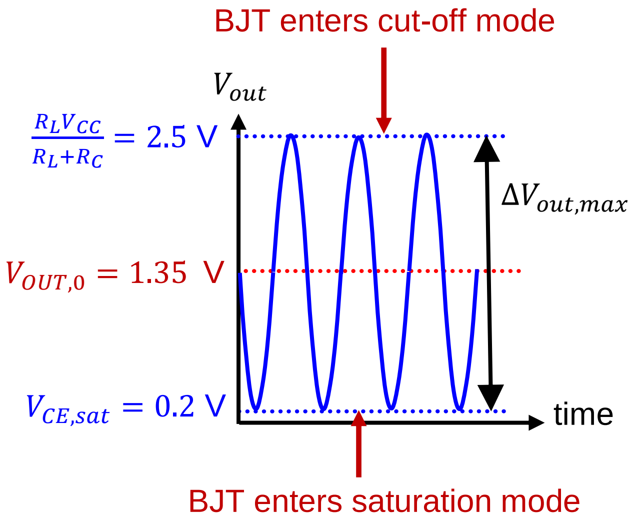
(f) The value of the DC bias voltage $V_{IN,0}$ must be chosen by the circuit designer so that the BJT operates in the middle of the forward-active mode, midway between the start of the cut-off mode and that of the saturation mode. This allows for maximum voltage and current swings at both input and output.

Let us focus first on the voltage swing issue. The "space" in which both voltages V_{in} and V_{out} can swing is that located between the start of the cut-off region and the start of the saturation region.

For our amplifier circuit, it is easy to see that the maximum value for the maximum output voltage swing, $\Delta V_{out,max}$, is given by $\frac{R_L V_{CC}}{R_L + R_C} - V_{CE,sat} = 2.3$ volts. No matter how well our circuit is designed, the value of $\Delta V_{out,max}$ cannot exceed $\frac{R_L V_{CC}}{R_L + R_C} - V_{CE,sat} = 2.3$ volts.

In the same way, the maximum value for the maximum input voltage swing, $\Delta V_{in,max}$, is given by $V_{in,sat} - V_{BE,on} = 0.46$ volts. In other words, the maximum peak-to-peak amplitude of the AC input voltage, v_{in}, that guarantees no distortion in v_{out} cannot exceed 0.46 volts.





The optimal value of $V_{IN,0}$ is located midway between the start of the cut-off region, corresponding to $V_{in} = V_{BE,on}$, and the start of the saturation region, corresponding to $V_{in} = V_{in,sat}$:

$$V_{IN,0} = \frac{V_{BE,on} + V_{in,sat}}{2} = \frac{0.7V + 1.16V}{2} = 0.93 \text{ volts.}$$

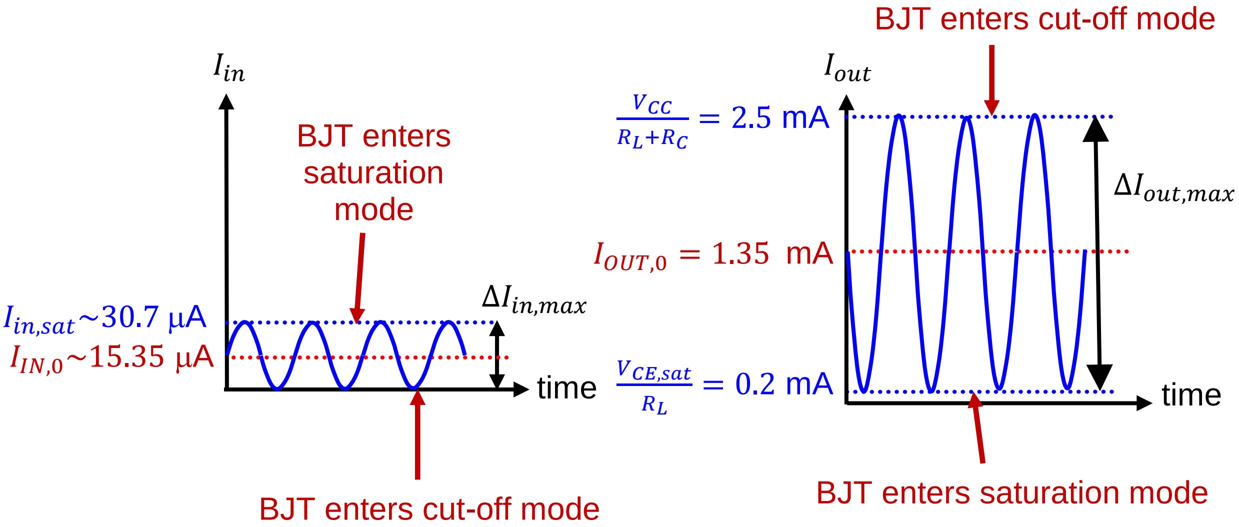
By choosing $V_{IN,0} = 0.93$ volts, the circuit designer ensures that both maximum voltage swings $\Delta V_{in,max}$ and $\Delta V_{out,max}$ are maximised: $\Delta V_{in,max} = V_{in,sat} - V_{BE,on} = 0.46$ volts and $\Delta V_{out,max} = \frac{R_L V_{CC}}{R_L + R_C} - V_{CE,sat} = 2.3$ volts.

(g) Let us now turn our attention to the current swing issue. The question that comes to mind is the following one: does a bias voltage value $V_{IN,0} = 0.93$ volts also maximise, by default, the maximum current swings $\Delta I_{in,max}$ and $\Delta I_{out,max}$?

The short answer is "yes". Let us prove that this is indeed the case.

The "space" in which the currents I_{in} and I_{out} are allowed to swing is that located between the start of the cut-off region and the start of the saturation region.

The maximum value for the maximum output current swing, $\Delta I_{out,max}$, is given by $\frac{V_{CC}}{R_L + R_C} - \frac{V_{CE,sat}}{R_L} = 2.3$ mA.



The maximum value for the maximum input current swing, $\Delta I_{in,max}$, is given by $I_{in,sat} \sim 30.7$ μA. In other words, the maximum peak-to-peak amplitude of the AC input current, i_{in}, that guarantees no distortion in i_{out} cannot exceed 30.7 μA.

The optimal value of $I_{IN,0}$ is located midway between the start of the cut-off region, corresponding to $I_{in} = 0$, and the start of the saturation region, corresponding to $I_{in} = I_{in,sat}$:

$$I_{IN,0} = \frac{0 + I_{in,sat}}{2} \sim \frac{30.7\mu A}{2} = 15.35 \text{ μA}.$$

We have computed the value that $I_{IN,0}$ must have for optimal biasing. The question now becomes the following one: does a DC bias voltage value $V_{IN,0} = 0.93$ volts lead, by default, to $I_{IN,0} \sim 15.35$ μA?

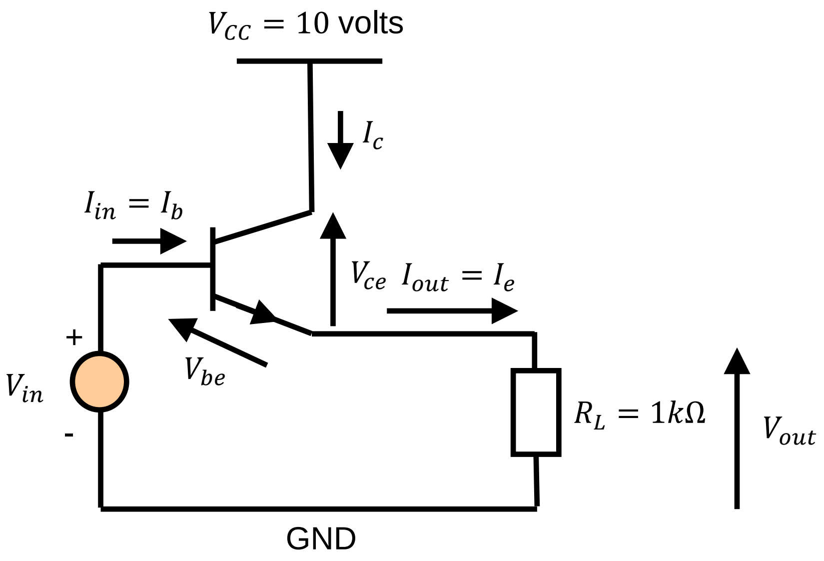
This question can be answered by remembering that $V_{IN,0}$ and $I_{IN,0}$ are linked through the expression $I_{IN,0} = \frac{V_{IN,0} - V_{BE,on}}{R_B}$. Hence, with $V_{IN,0} = 0.93$ volts, we obtain by default $I_{IN,0} = \frac{0.93V - 0.7V}{15k\Omega} \sim 15.33$ μA, which is exactly the optimal value for the biasing current.

We conclude that choosing the optimal value of $V_{IN,0}$ for voltage biasing also leads to an optimal current biasing. This is reassuring indeed.

As a conclusion, we have shown that the common-emitter circuit studied throughout this question is a linear voltage amplifier with a voltage gain $A_v = -5$ and also a linear current amplifier with a current gain $A_i = -75$.

We have also determined the value of the DC bias voltage, $V_{IN,0} = 0.93$ volts, that maximises the maximum input/output voltage swing as well as the maximum input/output current swing.

Question 3



(a) Let us write the general equations for this circuit, using Kirchhoff voltage law (KVL) and Ohm's law:

(1) Using KVL, $V_{CC} = V_{ce} + V_{out}$;

(2) Using KVL, $V_{be} + V_{out} = V_{in}$;

(3) Using Ohm's law, $V_{out} = R_L \cdot I_{out}$.

We obtain a set of three equations that are always valid, regardless the mode in which the BJT operates. Now, we can move to the next step. Consider the three possible modes of operation one at a time and examine what happens to the general equations.

(b) First mode of operation: Cut-off mode if $V_{be} < V_{BE,on}$ and $I_{in} = I_b = I_c = I_e = I_{out} = 0$.

In this mode, Equation 3, $V_{out} = R_L \cdot I_{out}$, leads to $V_{out} = 0$ because $I_{out} = 0$.

Equation 1, $V_{CC} = V_{ce} + V_{out}$, then becomes $V_{CC} = V_{ce}$ because $V_{out} = 0$.

Equation 2, $V_{be} + V_{out} = V_{in}$, becomes $V_{be} = V_{in}$ because $V_{out} = 0$. This result yields $V_{in} < V_{BE,on}$ as $V_{be} < V_{BE,on}$.

The inequality $V_{in} < V_{BE,on}$ provides us with the condition that the input voltage V_{in} must satisfy for the BJT to be cut-off.

Conclusion: when $V_{in} < V_{BE,on}$, the BJT is cut-off and we have $V_{out} = 0$ volt. As for the currents, we can write $I_{in} = I_{out} = 0$.

(c) Second mode of operation: Forward-active mode if $V_{be} = V_{BE,on}$ and $V_{ce} > V_{CE,sat}$. In this case, we have $I_e = (\beta_F + 1) \cdot I_b$, i.e., $I_{out} = (\beta_F + 1) \cdot I_{in}$, $I_{in} > 0$, and $I_{out} > 0$.

In this mode, Equation 3, $V_{out} = R_L \cdot I_{out}$, leads to $V_{out} > 0$ because $I_{out} > 0$.

Equation 1, $V_{CC} = V_{ce} + V_{out}$, leads to $V_{out} = V_{CC} - V_{ce} < V_{CC} - V_{CE,sat}$ as $V_{ce} > V_{CE,sat}$.

Equation 2, $V_{be} + V_{out} = V_{in}$, becomes $V_{out} = V_{in} - V_{BE,on}$ because $V_{be} = V_{BE,on}$. This result also yields $V_{in} > V_{BE,on}$ as $V_{out} > 0$.

We have obtained two conditions that need to be satisfied for the BJT to be forward active: $V_{in} > V_{BE,on}$ and $V_{out} < V_{CC} - V_{CE,sat}$. The second condition is not useful as such because what we need are conditions on the input voltage V_{in} only.

Since we now know that $V_{out} = V_{in} - V_{BE,on}$, we can replace the condition $V_{out} < V_{CC} - V_{CE,sat}$ with another condition $V_{in} - V_{BE,on} < V_{CC} - V_{CE,sat}$, which is equivalent to

$$V_{in} < V_{CC} + V_{BE,on} - V_{CE,sat}.$$

Remarkably, this last condition is, by default, always satisfied because the input voltage V_{in} cannot exceed V_{CC} (as the supply voltage is always the highest voltage in the circuit). We indeed know that $V_{CC} + V_{BE,on} - V_{CE,sat} > V_{CC}$. So, since $V_{in} < V_{CC}$, this directly implies that we always have $V_{in} < V_{CC} + V_{BE,on} - V_{CE,sat}$.

Consequently, we are left with the following condition that must be satisfied for the BJT to be forward active: $V_{BE,on} < V_{in} < V_{CC}$.

To be able to plot the DC current transfer characteristic later on, we need to replace this condition on the input voltage V_{in} with a condition on the input current I_{in}.

To do so, we have to express I_{in} as a function of V_{in}. This can be done as follows: the expression $V_{out} = V_{in} - V_{BE,on}$ is equivalent to $V_{in} = V_{out} + V_{BE,on} = R_L \cdot I_{out} + V_{BE,on} = R_L \cdot (\beta_F + 1) \cdot I_{in} + V_{BE,on}$, which yields

$$I_{in} = \frac{V_{in} - V_{BE,on}}{R_L(\beta_F + 1)}.$$

The condition $V_{BE,on} < V_{in} < V_{CC}$ thus becomes

$$\frac{V_{BE,on} - V_{BE,on}}{R_L(\beta_F + 1)} < \frac{V_{in} - V_{BE,on}}{R_L(\beta_F + 1)} < \frac{V_{CC} - V_{BE,on}}{R_L(\beta_F + 1)},$$

which is equivalent to $0 < I_{in} < \frac{V_{CC} - V_{BE,on}}{R_L(\beta_F + 1)}$.

Conclusion: when $V_{BE,on} < V_{in} < V_{CC}$, the BJT is forward active and we have $V_{out} = V_{in} - V_{BE,on}$. Alternatively, we can write that, when $0 < I_{in} < \frac{V_{CC} - V_{BE,on}}{R_L(\beta_F + 1)}$, the BJT is forward active and we have $I_{out} = (\beta_F + 1) \cdot I_{in}$.

(d) Third mode of operation: Saturation mode if $V_{be} = V_{BE,on}$ and $V_{ce} = V_{CE,sat}$. In this case, we have $I_{in} > 0$, and $I_{out} > 0$.

In this mode, Equation 3, $V_{out} = R_L \cdot I_{out}$, leads to $V_{out} > 0$ because $I_{out} > 0$.

Equation 1, $V_{CC} = V_{ce} + V_{out}$, becomes $V_{out} = V_{CC} - V_{CE,sat}$ because $V_{ce} = V_{CE,sat}$.

Equation 2, $V_{be} + V_{out} = V_{in}$, becomes $V_{out} = V_{in} - V_{BE,on}$ because $V_{be} = V_{BE,on}$. This result also yields $V_{in} > V_{BE,on}$ as $V_{out} > 0$.

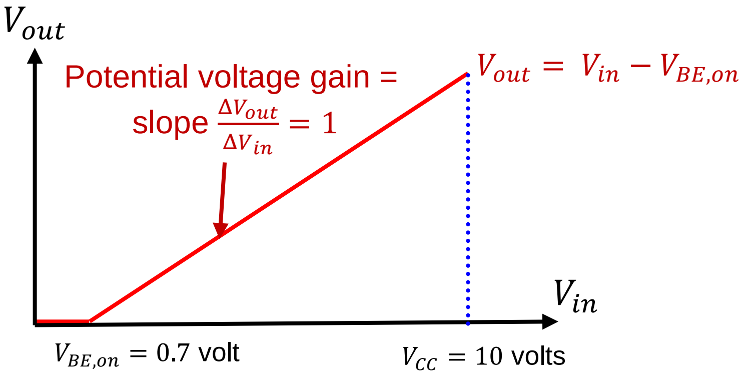
Here, there is clearly an issue because we obtain two different expressions for the output voltage: $V_{out} = V_{CC} - V_{CE,sat}$ and $V_{out} = V_{in} - V_{BE,on}$.

We thus reach a contradiction unless we have $V_{in} = V_{CC} + V_{BE,on} - V_{CE,sat}$. The latter equality can, in fact, never be realised because we remember that the maximum value of V_{in} in the circuit is the supply voltage V_{CC} and, as it turns out, the term $V_{CC} + V_{BE,on} - V_{CE,sat}$ is greater than V_{CC}.

We conclude that the BJT can never be saturated simply because this would lead to incompatible results. This does simplify our analysis tremendously.

(e) Let us now plot the DC transfer characteristics of the common-collector circuit by using the results that we have just obtained.

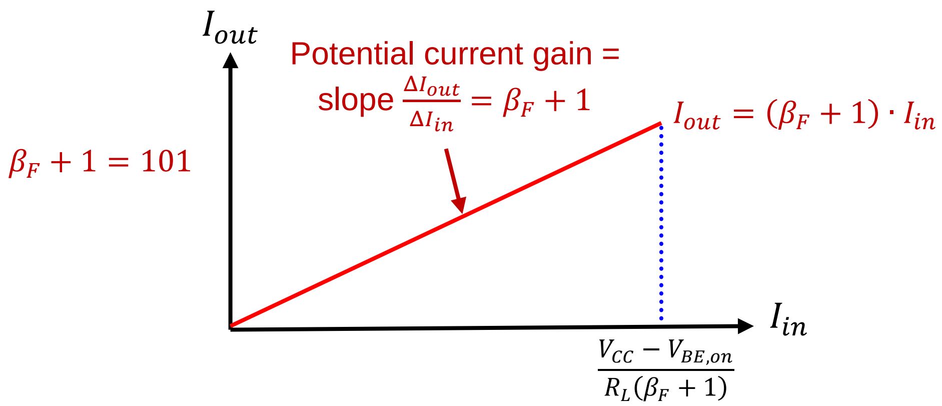
The DC voltage transfer characteristic is shown below.



It is observed that the common-collector circuit does NOT have the potential to be a voltage amplifier because the slope $\frac{\Delta V_{out}}{\Delta V_{in}}$ of the DC voltage transfer characteristic in the forward-active region is only equal to the unit.

(f) Let us now plot the DC current transfer characteristic of our common-collector amplifier.

The variation of I_{out} as a function of I_{in} is shown below.



We observe the potential of the common-collector circuit as a current amplifier because, in the forward-active region, the output current I_{out} is a linear function of the input current I_{in}:

$$I_{out} = (\beta_F + 1) \cdot I_{in}.$$

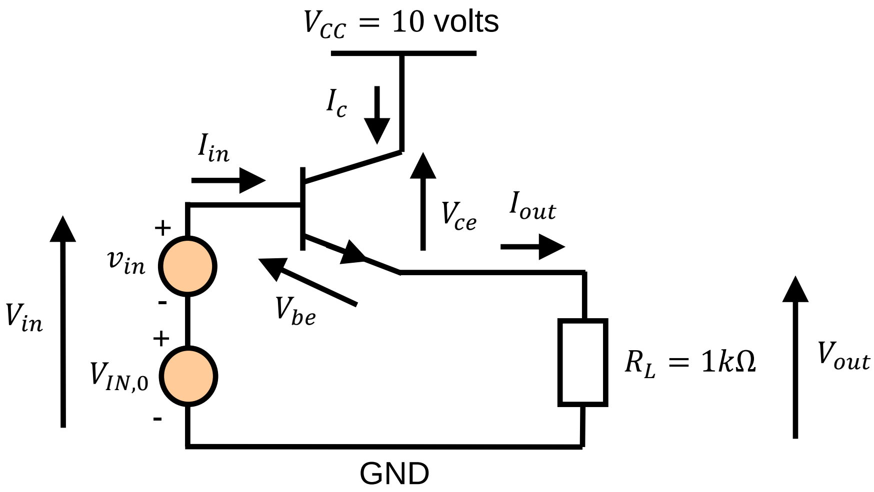
The current gain A_i of this amplifier would be the slope $\frac{\Delta I_{out}}{\Delta I_{in}}$ of the DC current transfer characteristic in the forward-active region, i.e., we would have $A_i = \beta_F + 1 = 101$.

Not only this corresponds to a very high current gain, but this gain has the advantage of being independent from the load resistance R_L, unlike what we observed with the common-emitter circuit studied in Questions 1 and 2.

To make our circuit work as a linear amplifier in practice, we will have to make sure that the BJT remains at all times in the forward-active mode and enters neither saturation nor cut-off mode. This issue is going to be addressed in Question 4.

Question 4

We can modify the circuit studied in Question 3 in order to design a linear common-collector amplifier that can amplify a current. The modified circuit is shown in the figure below.



The input voltage V_{in} is now composed of two voltage sources $V_{IN,0}$ and v_{in} connected in series. Hence, we can write $V_{in} = V_{IN,0} + v_{in}$.

The DC voltage source $V_{IN,0}$ is used to bias the circuit so that the BJT operates at all times in the forward-active mode. The quantity $V_{IN,0}$ is referred to as bias voltage and its value has to be chosen very carefully by the designer. Remember that a BJT has the ability to amplify a current or voltage only when it operates in the forward-active mode.

The AC voltage source v_{in} generates the AC voltage. It is perfectly reasonable to assume that v_{in} is a pure AC voltage signal with zero mean and small-amplitude symmetrical swings around zero volt.

Throughout this question, we are going to assume that the BJT operates in the forward-active mode. In other words, we are going to assume that the condition $V_{BE,on} < V_{in} < V_{CC}$ remains satisfied at all times.

(a) Since we have assumed that the BJT operates at all times in the forward-active mode, we can write

$$V_{in} = V_{BE,on} + V_{out} = V_{BE,on} + R_L \cdot I_{out} = V_{BE,on} + R_L \cdot (\beta_F + 1) \cdot I_{in},$$

which can be written as

$$I_{in} = \frac{V_{in} - V_{BE,on}}{R_L(\beta_F+1)} = \frac{V_{IN,0} + v_{in} - V_{BE,on}}{R_L(\beta_F+1)} = \frac{V_{IN,0} - V_{BE,on}}{R_L(\beta_F+1)} + \frac{v_{in}}{R_L(\beta_F+1)} = I_{IN,0} + i_{in}.$$

This expression indicates that the input current, I_{in}, can be written as the sum of a DC current, given by $I_{IN,0} = \frac{V_{IN,0} - V_{BE,on}}{R_L(\beta_F + 1)}$, and an AC current, given by $i_{in} = \frac{v_{in}}{R_L \cdot (\beta_F + 1)}$. So, we have $I_{in} = I_{IN,0} + i_{in}$.

(b) The results obtained in Question 3 can be applied to our circuit. In Question 3, we showed that, as long as the BJT operates in the forward active mode, i.e., if $V_{BE,on} < V_{in} < V_{CC}$, the output voltage V_{out} is given by $V_{out} = V_{in} - V_{BE,on}$.

So, we can write $V_{out} = V_{IN,0} + v_{in} - V_{BE,on}$.

The expression of V_{out} means that we can express the output voltage as follows:

$$V_{out} = V_{OUT,0} + v_{out},$$

where

- $V_{OUT,0} = V_{IN,0} - V_{BE,on}$ represents the DC component of V_{out};
- $v_{out} = v_{in}$ is a replica of the input AC voltage.

This result shows that the output voltage V_{out} is composed of a DC component, $V_{OUT,0}$, and an AC component, v_{out}. The AC component v_{out} is identical to the AC component v_{in} of the input voltage.

(c) The voltage gain of the circuit is defined as

$$A_v = \frac{v_{out}}{v_{in}},$$

and is thus given by $A_v = 1$.

This unit gain indicates that a common-collector amplifier does not amplify a voltage. In fact, it leaves the AC input voltage unchanged. This is a well-known feature of common-collector amplifiers.

(d) The results obtained in Question 3 can be applied to our circuit. In Question 3, we showed that, as long as the BJT operates in the forward active mode, i.e., if $0 < I_{in} < \frac{V_{CC}-V_{BE,on}}{R_L(\beta_F+1)}$, the output current I_{out} can be expressed as

$$I_{out} = (\beta_F + 1) \cdot I_{in}.$$

So, we can write $I_{out} = (\beta_F + 1) \cdot I_{IN0} + (\beta_F + 1) \cdot i_{in}$.

The expression of I_{out} means that we can express the output current as follows:

$$I_{out} = I_{OUT,0} + i_{out},$$

where

- $I_{OUT,0} = (\beta_F + 1) \cdot I_{IN0}$ represents the DC component of I_{out};
- $i_{out} = (\beta_F + 1) \cdot i_{in}$ is an amplified replica of the input AC current.

This result shows that the output current I_{out} is composed of a DC component, $I_{OUT,0}$, and an AC component, i_{out}. The AC component i_{out} is an amplified version of the AC component i_{in} of the input current.

(e) The current gain of the circuit is defined as

$$A_i = \frac{i_{out}}{i_{in}},$$

and is given by

$$A_i = \beta_F + 1 = 101,$$

since $\beta_F = 100$.

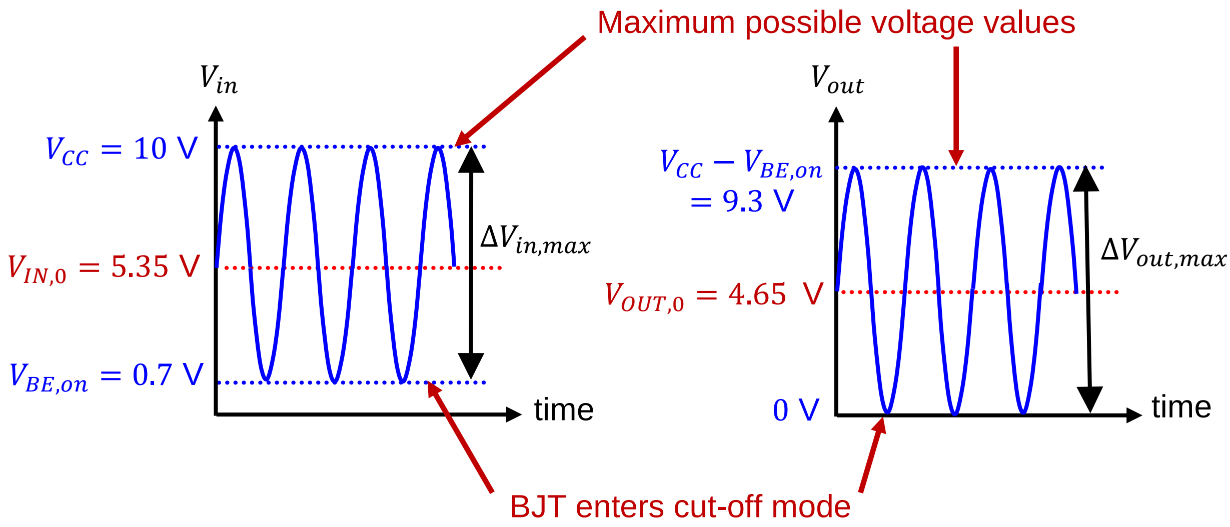
The results obtained in (c) and (e) show that our common-collector amplifier does not perform voltage amplification, but does achieve a large current gain $A_i = 101$ that has the advantage of being independent from the load resistance.

(f) The value of the DC bias voltage $V_{IN,0}$ should be chosen by the circuit designer so that the BJT operates in the middle of the forward-active mode, midway between the start of the cut-off mode and the inherent voltage limits set by the supply voltage V_{CC}. In fact, this allows for maximum voltage and current swings at both input and output.

Let us focus first on the voltage swing issue. The “space” in which both voltages V_{in} and V_{out} can swing is that located between the start of the cut-off region and the maximum voltages in the circuit corresponding to $V_{in} = V_{CC}$ and $V_{out} = V_{CC} - V_{BE,on}$.

For our common-collector circuit, it is easy to see that the maximum value for the maximum output voltage swing, $\Delta V_{out,max}$, is given by $V_{CC} - V_{BE,on} = 9.3$ volts. No matter how well our circuit is designed, the value of $\Delta V_{out,max}$ cannot exceed $V_{CC} - V_{BE,on} = 9.3$ volts.

In the same way, the maximum value for the maximum input voltage swing, $\Delta V_{in,max}$, is given by $V_{CC} - V_{BE,on} = 9.3$ volts. In other words, the maximum peak-to-peak amplitude of the AC input voltage, v_{in}, that guarantees no distortion in v_{out} cannot exceed 9.3 volts.



The optimal value of $V_{IN,0}$ is located midway between the start of the cut-off region, corresponding to $V_{in} = V_{BE,on}$, and the maximum input voltage in the circuit, corresponding to $V_{in} = V_{CC}$:

$$V_{IN,0} = \frac{V_{BE,on} + V_{CC}}{2} = \frac{0.7V + 10V}{2} = 5.35 \text{ volts.}$$

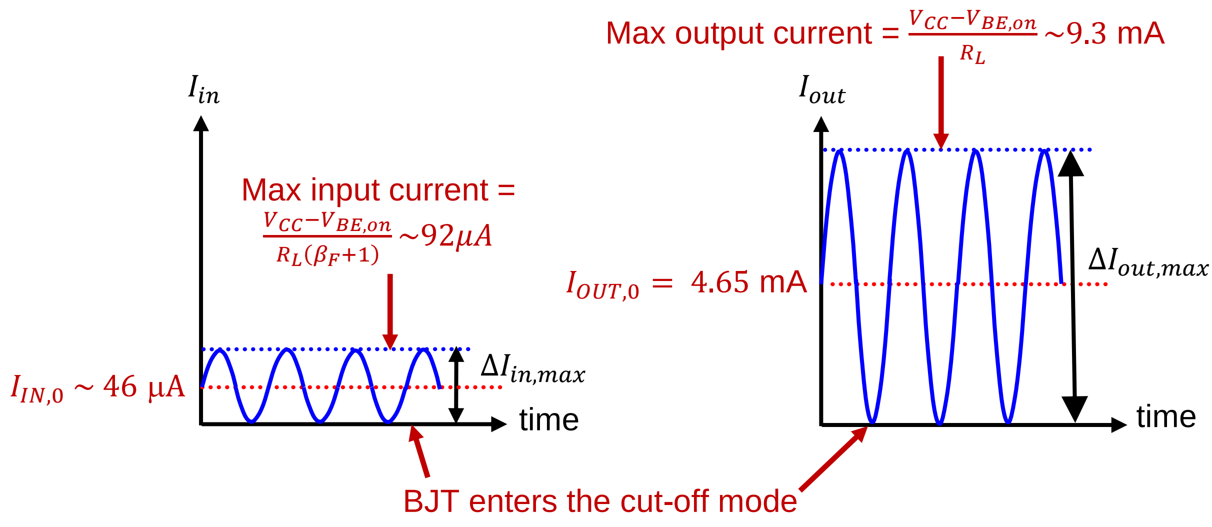
By choosing $V_{IN,0} = 5.35$ volts, the circuit designer ensures that both maximum voltage swings $\Delta V_{in,max}$ and $\Delta V_{out,max}$ are maximised: $\Delta V_{in,max} = \Delta V_{out,max} = V_{CC} - V_{BE,on} = 9.3$ volts.

(g) Let us now turn our attention to the current swing issue. The question that comes to mind is as follows: does a bias voltage value $V_{IN,0} = 5.35$ volts also maximise, by default, the maximum current swings $\Delta I_{in,max}$ and $\Delta I_{out,max}$?

The short answer is "yes". Let us prove that this is indeed the case.

The "space" in which the currents I_{in} and I_{out} are allowed to swing is that located between the start of the cut-off region and the maximum input and output current values that are inherently associated with $V_{in} = V_{CC}$ and $V_{out} = V_{CC} - V_{BE,on}$.

From the analysis previously carried out on our circuit, it is easy to show that the maximum input current value is given by $\frac{V_{CC}-V_{BE,on}}{R_L(\beta_F+1)} \sim 92$ μA, whereas the maximum output current value is given by $\frac{V_{CC}-V_{BE,on}}{R_L} \sim 9.3$ mA.



The maximum value for the maximum output current swing, $\Delta I_{out,max}$, is given by $\frac{V_{CC}-V_{BE,on}}{R_L} \sim 93$ mA.

The maximum value for the maximum input current swing, $\Delta I_{in,max}$, is given by $\frac{V_{CC}-V_{BE,on}}{R_L(\beta_F+1)} \sim 92$ μA. In other words, the maximum peak-to-peak amplitude of the AC input current, i_{in}, that guarantees no distortion in i_{out} cannot exceed 92 μA.

The optimal value of $I_{IN,0}$ is located midway between the start of the cut-off region, corresponding to $I_{in} = 0$, and the maximum input current given by $I_{in} = \frac{V_{CC} - V_{BE,on}}{R_L(\beta_F + 1)}$:

$$I_{IN,0} = \frac{0 + \frac{V_{CC} - V_{BE,on}}{R_L(\beta_F+1)}}{2} \sim \frac{92\mu A}{2} = 46\ \mu\text{A}.$$

We now know the value that $I_{IN,0}$ must have for optimal current biasing. The question becomes as follows: does a DC bias voltage value $V_{IN,0} = 5.35$ volts lead, by default, to $I_{IN,0} \sim 46\ \mu\text{A}$?

This question can be answered by remembering that $V_{IN,0}$ and $I_{IN,0}$ are linked through the expression $I_{IN,0} = \frac{V_{IN,0} - V_{BE,on}}{R_L(\beta_F + 1)}$. Hence, with $V_{IN,0} = 5.35$ volts, we obtain by default $I_{IN,0} = \frac{5.35V - 0.7V}{101k\Omega} \sim 46\ \mu\text{A}$, which is exactly the optimal value for the biasing current.

We conclude that choosing the optimal value of $V_{IN,0}$ for voltage biasing also leads to an optimal current biasing.

As a conclusion, we have shown that the common-collector circuit studied throughout this question is not a voltage amplifier as it only displays a voltage gain $A_v = 1$, but is an excellent current amplifier with a current gain $A_i = 101$.

Compared to the common-emitter amplifier studied in Question 2, the common-collector amplifier possesses the advantage of having a current gain that does not depend on the load resistance. In other words, this current gain is not affected by the type of downstream circuit connected to the amplifier output port. This is a tremendous advantage in practice.

We have also determined the value of the DC biasing voltage, $V_{IN,0} = 5.35$ volts, that maximises the maximum input/output voltage swing as well as the maximum input/output current swing.

- END -